In control system design, two parameters are most important in describing the dynamic properties of a transfer function, i.e.,

1. Steady state gain K: the steady state gain reflects the effect of the manipulated variable $u_j$to the controlled variable y_i
2. Average residence time $T_{ar_{ij}}$: the average residence time is accountable for the response speed of the controlled variable $y_i$to manipulated variable u_j.

To measure the interaction effects, the normalized gain $K_{N,ij}$ for a particular transfer function, $g_{ij}(s)$ is given as,

$$K_{N,ij} \triangleq \frac{k_{ij}}{T_{ar_{ij}}} = \frac{k_{ij}}{\tau_{ij}+\theta_{ij}} \text{ i, j=1,2,..} \quad (9)$$

For the whole system, it can be written in a matrix form as,

$$K_N = \begin{bmatrix} K_{N,11} & K_{N,12} \\ K_{N,21} & K_{N,22} \end{bmatrix} \quad (10)$$

Similar to RGA, the Normalized relative gain matrix can be defined between output variable $y_i$and input variableu_j,$\wedge_{ij}$, as the ratio of two normalized gains

$$\wedge_{N,ij} = \frac{K_{N,ij}}{\hat{K}_{N,ij}} \text{i, j=1,2,…} \quad (11)$$

where $\hat{K}_{N,ij}$ is the normalized gain between output variable $y_i$and input variable u_j when all other loops are closed.

The relative normalized gain array (RNGA)

$$\wedge_N = \begin{bmatrix} \wedge_{N,11} & \wedge_{N,12} \\ \wedge_{N,21} & \wedge_{N,22} \end{bmatrix} \quad (12)$$

can be calculated by

$$\wedge_N = K_N \otimes K_N^{-T} \quad (13)$$

where the operator ⊗ is the Hadamard product.

The relative normalized gain reflects the combined changes in both steady state gain and dynamic when all other loops are open and when all other loops are closed. To separate the two changes, first the relative average residence time γ_{ij}should be defined as the ratio of loop $y_i - u_j$ average residence time between when other loops are closed and when other are open, i.e.,

$$\gamma_{ij} \triangleq \frac{\hat{T}_{ar,ij}}{T_{ar,ij}} \text{ i, j =1,2,…} \quad (14)$$

Using the definition of RNGA, the equations can be written as,

$$\hat{k}_{ij} \times T_{ar_{ij}} = \frac{K_{ij} \times T_{ar_{ij}}}{\wedge_{N,ij}} \text{ i, j=1,2,.} \quad (15)$$

where $T_{ar_{ij}}$is the average residence time of loop *i-j* when other loops are closed. Equation (15) provides both gain and average residence time change information when all other loops are closed. To separate these two changes, the definition of RGA is used here,

$$\hat{k}_{ij} = \frac{k_{ij}}{\lambda_{ij}} \text{ i, j=1,2,…} \quad (16)$$

When the relative average times are calculated for all the input/output combinations of the TITO process, it results in an array of the form, i.e., relative average residence time array (RARTA) which is defined as

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \triangleq \wedge_N \odot \wedge \quad (17)$$

$$\Gamma = \begin{bmatrix} \wedge_{N,11} & \wedge_{N,12} \\ \wedge_{N,21} & \wedge_{N,22} \end{bmatrix} \odot \begin{bmatrix} \wedge_{11} & \wedge_{12} \\ \wedge_{21} & \wedge_{22} \end{bmatrix} \quad (18)$$

where the operator ⊙ is the hadamard division.

As the relative average time is the ratio of the average residence times between when other loops are closed and when other loops are open, $\hat{T}_{ar,ij}$represent the dynamic changes of the transfer function $g_{ij}(s)$when other loops closed. By the definition of RARTA,

$$\hat{T}_{ar,ij} = \gamma_{ij} \times T_{ar,ij} = \gamma_{ij} \times \tau_{ij} + \gamma_{ij} \times \theta_{ij} \quad (19)$$

The average residence time of loop *i-j* th when other loops are closed is the open loop average residence time scaled by a factorγ_{ij}. In process control, steady state gain, time constant and time delay are the parameters that are of topmost interest for control system design. By using RGA and RARTA information, gain and phase changes of a transfer function element when other loops closed can be uniquely determined, i.e., a transfer function element of a MIMO process when other loops are closed can be approximated by a transfer function element having the same form as the open-loop transfer function element, but the steady state gain, time constant and time are scaled by $\frac{1}{\lambda_{ij}}$and γ_{ij}, respectively, i.e.,

$$\hat{g}_{ij}(s) = \hat{k}_{ij} \times \frac{1}{\hat{\tau}_{ij}s+1} e^{-\hat{\theta}_{ij}s} = \frac{k_{ij}}{\lambda_{ij}} \times \frac{1}{\gamma_{ij}\tau_{ij}s+1} e^{-\gamma_{ij}\theta_{ij}s} \text{i,j=1,2,…} \quad (20)$$

V. DECENTRALIZED CONTROLLER DESIGN USING EQTF

Since EQTF have incorporated the information of loop interactions, the MIMO process can be decomposed into a set of SISO processes then decentralized PID controllers can be designed to stabilize these SISO loops independently. In application, however, it is desirable that the MIMO system remains stable if any of the loops is taken in or out of service. This requires that the controllers be designed conservatively. Such motivates the use of modified EQTF which keeps the same form of the EQTF but has parameters taking the larger values of EQTF and its corresponding open-loop transfer function,

$$\tilde{g}_{ij}(s) = \frac{\tilde{k}_{ij} e^{-\tilde{\theta}_{ij}s}}{(\tilde{\tau}_{ij}s+1)} \quad (21)$$

where $\tilde{g}_{ij}(s)$ is the modified EQTF in which

$$\tilde{k}_{ij} = max\{k_{ij}, \tilde{k}_{ij}\}, \quad \tilde{\tau}_{ij} = max\{\tau_{ij}, \tilde{\tau}_{ij}\}, \quad \tilde{\theta}_{ij} = max\{\theta_{ij}, \tilde{\theta}_{ij}\} \quad (22)$$

That the larger parameters usually imply the more challenging situations for control, which sequentially implies that the controller design, will be more conservative as compared to using the smaller parameters. As each controller design becomes a SISO case, any good PID tuning methods may apply. The simple internal model control (SIMC) tuning method is adopted for simplicity and robustness. The SIMC controller settings for the first-order with time delay process [14] is given as

$$g(s) = k \frac{e^{-\theta s}}{(T_I s+1)} \quad (23)$$