Applications of Fuzzy Sets in the Area of Group Decision for N - Person Game

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***Abstract* – Applications of fuzzy sets within the field of decision making were based on the extensions of the classical theories of decision making. Decision making may include any choice or selection of alternatives and is therefore of prime importance in many fields of science. Many managerial decisions are however made with some uncertainty. In case of decision making under conditions of risk or uncertainty more than one states of nature exists, and probabilities or fuzzy decision theories are used to deal with fuzziness. Therefore the quality of the decisions may get diverted from the actual decisional aspects. This paper proposes a set of guidelines based on fuzzy model to be applied for game theories and also work out fuzzy to crisp conversion on the analyzed group decision sets.**

***Keywords:* Decision Alternative, Fuzzy Sets, Uncertainty**

I. Introduction

Fuzziness can be introduced at several points in the existing models of decision making. In case of uncertainties concerning the states of the system, the situation can be handled by modeling the states as well as the utility assigned to each state with fuzzy sets [3]. Fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. The design of a fuzzy set is not trivial and several approaches (Bellman and Zadeh, 1970) have been suggested to accommodate certain constraints C and objectives G in a decision making environment. Here both constraints and objectives are treated as fuzzy sets characterized by the following membership functions.

$$\mu_C \ : X \rightarrow [0, 1] \text{ and} \tag{1}$$

$$\mu_G \ : X \rightarrow [\,0, 1] \tag{2}$$

where X is the universal set of alternative actions.

II. Fuzzy Application In Game Theory

Group decision happens when two or more individuals or organizations with conflicting objectives try to make decisions [2]. When decisions which are made by more than individual are modeled the differences from the perspective of an individual decision maker should be considered. Theories such as game theory, team theory and group decision theory deals with these different perspectives.

A. Fuzzy Model for N-Person Game

Fuzzy model for n-person game is an extension of the model proposed by Blin (1974) for group decision. To take an appropriate decision it is necessary to evaluate to what extent the objective is satisfied by the constraint set. In order to do this it is assumed that each member of a group of n decision makers is assumed to have a reflexive, anti symmetric and transitive preference ordering. As n individual decision makers are involved in decision making the membership grade function is formed by considering the degree of preference of one alternative over another. Thus using the majority scheme

$$S(x_i, x_j) = \frac{N(x_i, x_j)}{n} \tag{3}$$

where x_i, x_j are the alternative preferences, N is the number of persons preferring the alternatives and n is the total number of decisions.

The constraint and objective set corresponding to the set of n decision makers can be expressed as

$$\mu_{Oi}\,(d) = \mu_{Oi'}\,(X_i(d)) \tag{4}$$

$$_{Cj}(d) = \mu_{Cj'}\,(C_j(d)) \tag{5}$$

where $X_i : d \rightarrow X_i$

$C_j : d \rightarrow Y_j$

and d is the set of decision alternatives which are indicated

based on the degree of group preference. These decision alternatives are to be aggregated in such a way that it represents a fuzzy relationship. One method of aggregating is to include the possibility of decision alternative, such that X_i (x_i, x_j) = 1 if and only if the preference order of one individual decision alternative x_i has complete control over the other decision alternatives.

III. Analysis and Interpretations

Based on the consideration of individual preferences a situation is analyzed here. Suppose a goods manufacturer wishes to decide among two products to bring out in the market and what level of advertising to use. Assume that S_1, S_2 and S_3 represents high, average and low demands for the products and A_1, A_2 and A_3 represents the low, medium and high expenditure advertising programme. A possibility of 6 alternative decisions is considered with a set of 3 advertising alternatives as follows:

$D_1 = \{ A_1, A_2, A_3\}$

$D_2 = \{ A_3, A_1, A_2\}$

$D_3 = \{ A_2, A_1, A_3\}$

$D_4 = D_5 = \{ A_1, A_3, A_2\}$

$D_6 = \{ A_2, A_3, A_1\}$

By equation (1) for the ordering of 6 decision alternative the following ordering relation is obtained.

$$\begin{array}{c|ccc|} & A_1 & A_2 & A_3 \\ A_1 & 0 & 0.67 & 0.67 \\ A_2 & 0.33 & 0 & 0.33 \\ A_3 & 0.33 & 0.5 & 0 \end{array}$$

Fig. 1 Ordering Relation

IV. Fuzzy to Crisp Conversion

Though the real world data may be fuzzy in nature, the process of applying the decision criteria requires selecting an optimal decision. This optimal decision is implemented in the machine in a crisp way. Thus there is a necessity to make a conversion from fuzzy alternative to crisp form. There are many methods in the literature for the process of fuzzy to crisp conversion. As the fuzzy model is developed for the n-person game the method that uses the mean or centre of gravity of fuzzy sets may be used. Thus the different decision alternatives may lead to various preference ordering of relations. The decision order corresponding to the preference relation $D_{0.5} = \{ (A_3, A_2) \}$ is chosen for fuzzy to crisp conversion. Thus the value 0.5 represents the decision alternative based on group preference.

V. Conclusion

This paper proposes some guidelines that can be used for decision making under conditions of uncertainty where there is scope for alternative decision criteria. Exploration of fuzzy data sets in the area of game theory corresponding to problems with dominance is to be further investigated.

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