

Prediction of Effective Thermal Conductivity of Cu/Solder System by Using Interfacial Layer in Two Phase System

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Abstract - A theoretical model is presented to predict effective thermal conductivity of real two phase materials from the values of thermal conductivity of the constituent phases and their volume fraction. Here we develop a numerical expression for effective thermal conductivity of Cu-solder system in two phase system is being comprised of contributions from both the phases with interfacial layer. A new correlation term F is introduced for highly conducting phase, non-linear flow of heat flux lines and random distribution of the phases. Here, we get best fitting relation for F. Comparison of the proposed relation with different models has also been made. The values predicted by the proposed model are in close and good agreement with experimental values and more accurate than those obtained from existing models.

Keywords: Effective thermal conductivity (ETC), real two phase system, correction term, interfacial layer, resistor model, metal foam.

I. INTRODUCTION

An ever increasing interest has been focused on heat and mass transfer processes in porous media due to their growing importance in functional material design, thermal managements of micro systems, and even in bio-medical engineering [1-6]. Among them the high-porosity foam materials are novel types of industrial materials with low density and unique transport properties, different from those of conventional porous media, which bring them to special and important applications. For examples, the metal and ceramic foams have been used in design of aircraft wing structure in the aerospace industry, in catalytic surfaces for chemical reactions, as the core structure for high strength panels, and the containment matrix and burn rate enhancer for solid propellants. All these applications require accurate evaluations of the effective thermal conductivity of such porous foam metal. The complexity of geometry encountered in the metal foams, along with the large difference in thermal conductivity of the constituents make it difficult to predict the ETC of metal foams using previous work done [1-6].

Heat conduction in porous matrices has been studied in detail by many researchers and has been summarized in a number of excellent review articles [7-12]. Recently, Calmada and Mahajan [13]; Boomsma and Poulikakos [14]; and R. Singh and H.S. Kasana [15] independently developed models utilizing geometrical estimate for calculation of ETC for metallic foams saturated with a fluid. For high porosity metal foams Calmada and Mahajan [13] presented a one dimensional heat conduction model considering the porous medium to

be formed of a two dimensional array of hexagonal cells. Whereas Boomsma and Poulikakos [14] proposed a three dimensional model using metal foam structure in the form of tetrakaidecahedron cells with cubic nodes at the intersection of two fibers. In similar manner R. Singh and H.S. Kasana [15] presented a resistor model for two phase system. All the models involved a geometric parameter that was evaluated using the experimental data.

II. THEORY & MATHEMATICAL FORMULATION

In this model, we want to develop an empirical relation for quick estimation of ETC of highly porous systems. In order to incorporate varying individual geometries and non-linear flow of heat flux lines generated by the different in thermal conductivity of the constituent phases, a correlation term F has been introduced. Our approach is simpler and provides wider applicability of the proposed relation and enhances its ability to predict correctly the ETC of highly porous metal foams. Considering various components as resistors one can take a combination of such resistors to predict ETC. This is a common practice adopted to predict ETC from the thermal conductivity of the constituent phases. Accepting the similarity, a relation is proposed here in the following manner.

Consider a two-phase medium made up of solid material (subscript s); a fluid (subscript f) and interfacial layer between solid and fluid (subscript sf) filling the pore space having volume fractions ϕ_s , ϕ_f and ϕ_{sf} respectively. The matrix is supposed to be made up of layers oriented parallel and perpendicular to the direction of heat flow, alternately as depicted in Fig. 1.

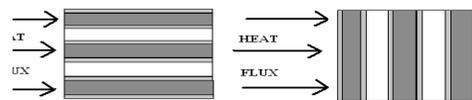


Fig. 1 Configuration of the resistors in a two-phase system

The thermal conductivity of parallel layers $\lambda_{||}$ is given by the weighted arithmetic mean and that of perpendicular layers λ_{\perp} by weighted harmonic mean. The corresponding expressions are

$$\lambda_{||} = \phi_f \lambda_f + \phi_s \lambda_s + \phi_{sf} \lambda_{sf} \quad (1)$$

$$\lambda_{\perp} = \frac{\lambda_f \lambda_s \lambda_{sf}}{\phi_f \lambda_s \lambda_{sf} + \phi_s \lambda_f \lambda_{sf} + \phi_{sf} \lambda_s \lambda_f} \quad (2)$$

Where λ_s = thermal conductivity of solid phase, λ_f = thermal conductivity of fluid phase, λ_{sf} = thermal conductivity of interfacial layer, ϕ_s = volume fraction of solid phase, ϕ_f = volume fraction of fluid phase, ϕ_{sf} = volume fraction of interfacial layer.

The λ_{\parallel} and λ_{\perp} are the upper and lower bounds on the ETC of a two phase system, respectively, therefore, ETC will obviously lie between these two bounds. As these relations do not predict the ETC of a real two-phase system correctly, a different kind of weighted geometric mean is proposed as

$$\lambda_{\epsilon} = (\lambda_{\parallel}^F \lambda_{\perp}^{(1-F)}), 0 \leq F \leq 1 \quad (3)$$

Where Fth fraction of the material is oriented in the direction of heat flow and remaining (1 - F)th fraction is oriented in the perpendicular direction. The ETC of a two-phase system is found to depend on λ_s / λ_f of the constituent phases. A higher ratio favours a larger fraction of the thermal conductivity in a direction perpendicular to heat flow.

Eq. (3) is solved for F in terms of λ_{\parallel} , λ_{\perp} and λ_{ϵ} . The solution is

$$F = \frac{\ln \left[\phi_f \frac{\lambda_{\epsilon}}{\lambda_f} + \phi_s \frac{\lambda_{\epsilon}}{\lambda_s} + \phi_{sf} \frac{\lambda_{\epsilon}}{\lambda_{sf}} \right]}{\ln \left[\phi_f^2 + \phi_s^2 + \phi_{sf}^2 + \phi_f \phi_s \left(\frac{\lambda_f}{\lambda_s} + \frac{\lambda_s}{\lambda_f} \right) + \phi_f \phi_{sf} \left(\frac{\lambda_f}{\lambda_{sf}} + \frac{\lambda_{sf}}{\lambda_f} \right) + \phi_s \phi_{sf} \left(\frac{\lambda_s}{\lambda_{sf}} + \frac{\lambda_{sf}}{\lambda_s} \right) \right]} \quad (4)$$

The correlation term F be a function of ratio of thermal conductivity of the constituent phases and porosity of the system. Keeping this in mind, we have tried many combinations. One such plot of F versus $R = \ln(\phi \lambda_s / \lambda_f)$ is shown in fig. 2, which is the best suited one for our proposed model. It is observed from the graph that F increases roughly linearly with increasing R. we have used a curve fitting technique and found that the expression

$$F = -0.351R + 0.852 \quad (5)$$

$$F = -0.351 \ln \left(\phi_f \frac{\lambda_s}{\lambda_f} \right) + 0.852 \quad (6)$$

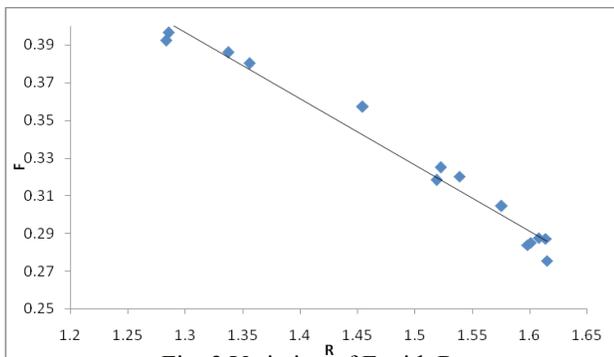


Fig. 2 Variation of F with R.

Best fitted the curve in Fig. 2. It is also observed from experimental results that the expression (5) or (6) represent the true state of affairs of a real system. F is calculated using Eq. (6).

In the present model, $\lambda_{sf} = 2 \lambda_f$ (i.e. $k_{layer} = 2 k_f$ the same value as that given by Leong et.al. [18]) is used in the calculation of the thermal conductivity. As we know that the volume fraction of a system is one. So we get the volume fraction of solid, fluid and interfacial layer is equal to one i.e. $\phi_s + \phi_f + \phi_{sf} = 1$. According to this phenomena we can easily calculate the volume fraction of interfacial layer (ϕ_{sf}), and we get a very small range for interfacial layer. So the volume fraction of interfacial layer is just 0.001 for Cu/solder system.

III. RESULTS AND DISCUSSION

To validate our empirical correlation, we considered one representative combinations of high porosity foams in our computations i.e. Cu-Solder[17,19]. The values of ETC has been calculated using Eq. (3) and compared the results as shown in Table 1. Table 1 for Cu/Solder system $\lambda_s = 398$, $\lambda_f = 78.1$, $\lambda_{sf} = 2\lambda_f = 156.2$, $\phi_{sf} = 0.001$ and other parameters given in table.

In Fig. 3, experimental results of the ETC for the same samples are shown with computed values using Eq. (3) and with the models [15,16]. It is found that ETC calculated using Eq. (3), gives closer results then R. Singh et.al. model [19] and is comparable with Bhattacharya et.al.model [16]. Comparing our correlation using Eq. (3) to the experimental data shows that our model follows the curve of the data points very well as in Fig. 3. The average deviation from experimental values for the models has also been computed, it is found that R. Singh et.al. model has 3.66% error whereas Bhattacharya et.al.model has 10.42%. deviation is least for our empirical proposition when we use Eq. (3) in the calculation of ETC. The average error between calculated values of ETC with the experimental results is 0.11%.

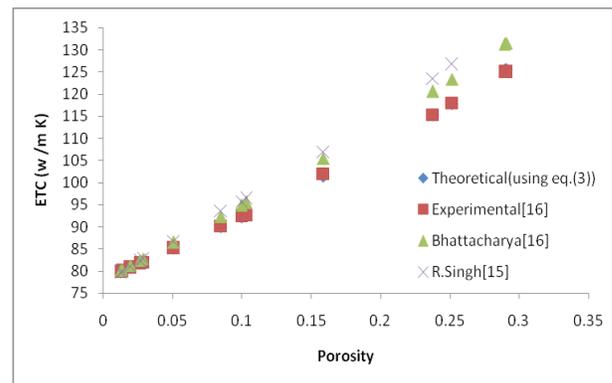


Fig. 5 Variation of ETC with porosity in Cu-solder system

TABLE I COMPARISON OF ETC VALUES FOR Cu/SOLDES SYSTEM IN TWO-PHASE METAL FOAMS CALCULATED USING EQ. (3)

S.No.	ϕ_s	$\lambda_e(\text{exp})$	$\lambda_e(\text{th})$	%Error
1	0.0124	79.8	79.8	0
2	0.0136	80	79.9	0
3	0.0507	85.2	85.1	0
4	0.0996	92.4	92.2	0.1
5	0.0195	80.8	80.8	0
6	0.0263	81.7	81.7	0
7	0.0286	82	82	0
8	0.1029	92.7	92.7	0
9	0.2377	115.4	115.2	0.1
10	0.0848	90.2	90	0.1
11	0.1586	102	101.4	0.5
12	0.2516	118	117.8	0.1
13	0.2894	125	125.3	0.2
14	0.291	125	125.6	0.5

Average deviation = 0.11%

The thermal conductivity is in W / m K. Here $\lambda_e(\text{exp})$ and $\lambda_e(\text{th})$ denote the experimental and theoretical values of λ_e , respectively. The $\lambda_e(\text{exp})$ has been taken from Bhattacharya et.al.model [16].

IV. CONCLUSIONS

The correlation presented here showed that the ETC strongly depends on porosity and the ratio of thermal conductivity of the constituents. Other factors have small effect on the ETC. The parameters of fluids, such as the size, volume fraction, the thickness of the interfacial layer, are shown to play important roles in the enhancement of thermal conductivity. The model predictions have been shown to be reasonable and are in good agreement with the available experimental data. It have two terms, correlation factor and weight factor, which are valid for metal foams with high porosity. The empirical model proposed here is capable of predicting ETC values closer to the experimental results for aluminum and RVC foams, using air and water as fluid media and similarly graphite and polyurethane foams using air as fluid media. It should be noted that experimental data for the ETC of metal foams with well characterized micro-structures having wide range of porosity are still in short supply. It is expected that the experimentally validated model will be helpful in the evaluation of the ETC for foam like materials in the whole range of porosity.

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