

On $M_X\alpha\psi(\mathcal{H})$ in terms of Minimal Structure Spaces

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Abstract - In this paper we introduce the notion of $M_X\alpha\psi$ -irresolute and $M_X\alpha\psi(\mathcal{H})$. Further, we derive some properties of $M_X\alpha\psi(\mathcal{H})$ and Pasting Lemma for $M_X\alpha\psi$ -irresolute functions.

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I. INTRODUCTION

In 1950, H. Maki et al. [6] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X -open set and m_X -closed set and characterize those sets using m_X -cl and m_X -int operators respectively. Further they introduced m -continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [1, 2, 3, 4, 7, 8, 11, 12].

The notion of $M_X\alpha\psi$ -closed set and $M_X\alpha\psi$ -continuous function were introduced and studied by M. Parimala [10]. In this paper we introduce $M_X\alpha\psi$ -irresolute and $M_X\alpha\psi(\mathcal{H})$. Further, we obtain some characterizations and properties.

II. PRELIMINARIES

In this section, we introduce the \mathcal{M} -structure and define some important subsets associated to the \mathcal{M} -structure and the relation between them.

Definition 2.1 [6] Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X . Where m_X is an \mathcal{M} -structure (or a minimal structure) on X , if φ and X belong to m_X .

The members of the minimal structure m_X are called m_X -open sets, and the pair (X, m_X) is called an m -space. The complement of m_X -open set is said to be m_X -closed.

Definition 2.2 [6] Let X be a nonempty set and m_X an \mathcal{M} -structure on X . For a subset A of X , m_X -closure of A and m_X -interior of A are defined as follows:

$$m_X\text{-cl}(A) = \bigcap \{F : A \subseteq F, X - F \in m_X\}$$

$$m_X\text{-int}(A) = \bigcup \{F : U \subseteq A, U \in m_X\}$$

Lemma 2.3 [6] Let X be a nonempty set and m_X an \mathcal{M} -structure on X . For subsets A and B of X , the following properties hold:

- $m_X\text{-cl}(X - A) = X - m_X\text{-int}(A)$ and $m_X\text{-int}(X - A) = X - m_X\text{-cl}(A)$.
- If $X - A \in m_X$, then $m_X\text{-cl}(A) = A$ and if $A \in m_X$ then $m_X\text{-int}(A) = A$.
- $m_X\text{-cl}(\varphi) = \varphi$, $m_X\text{-cl}(X) = X$, $m_X\text{-int}(\varphi) = \varphi$ and $m_X\text{-int}(X) = X$.
- If $A \subseteq B$ then $m_X\text{-cl}(A) \subseteq m_X\text{-cl}(B)$ and $m_X\text{-int}(A) \subseteq m_X\text{-int}(B)$.
- $A \subseteq m_X\text{-cl}(A)$ and $m_X\text{-int}(A) \subseteq A$.
- $m_X\text{-cl}(m_X\text{-cl}(A)) = m_X\text{-cl}(A)$ and $m_X\text{-int}(m_X\text{-int}(A)) = m_X\text{-int}(A)$.
- $m_X\text{-int}(A \cap B) = (m_X\text{-int}(A)) \cap (m_X\text{-int}(B))$ and $(m_X\text{-int}(A)) \cup (m_X\text{-int}(B)) \subseteq m_X\text{-int}(A \cup B)$.
- $m_X\text{-cl}(A \cup B) = (m_X\text{-cl}(A)) \cup (m_X\text{-cl}(B))$ and $m_X\text{-cl}(A \cap B) \subseteq (m_X\text{-cl}(A)) \cap (m_X\text{-cl}(B))$.

Lemma 2.4 [11] Let (X, m_X) be an m -space and A a subset of X . Then $x \in m_X\text{-cl}(A)$ if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x .

Definition 2.5 [8] A minimal structure m_X on a nonempty set X is said to have the property \mathcal{B} if the union of any family of subsets belonging to m_X belongs to m_X .

Remark 2.6 [5] A minimal structure m_X with the property \mathcal{B} coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7 [2] Let X be a nonempty set and m_X an \mathcal{M} -structure on X satisfying the property \mathcal{B} . For a subset A of X , the following property hold:

- $A \in m_X$ iff $m_X\text{-int}(A) = A$
- $A \in m_X$ iff $m_X\text{-cl}(A) = A$
- $m_X\text{-int}(A) \in m_X$ and $m_X\text{-cl}(A) \in m_X$.

Definition 2.8. A subset A of an m -space (X, m_X) is called

- (a) $m_X\alpha$ -open set [10] if $A \subseteq m_X \text{int} (m_X \text{cl}(m_X \text{int}(A)))$ and an $m_X\alpha$ -closed set if $m_X \text{cl} (m_X \text{int}(m_X \text{cl}(A))) \subseteq A$.
- (b) m_X -pre open set [12] if $A \subseteq m_X \text{int} (m_X \text{cl}(A))$ and an m_X -pre closed set if $m_X \text{cl}(m_X \text{int}(A)) \subseteq A$.
- (c) m_X -semi open set [12] if $A \subseteq m_X \text{cl}(m_X \text{int}(A))$ and an m_X -semi closed set if $m_X \text{int}(m_X \text{cl}(A)) \subseteq A$.

Definition 2.8. A subset A of an m -space (X, m_X) is called

- (a) an m_X -semi generalized-closed [12] (briefly m_X -sg-closed) set if $m_X \text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X -semi-open in (X, m_X) . The complement of an m_X -sg-closed set is called an m_X -sg-open set.
- (b) an m_X - ψ -closed [10] set if $m_X \text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X -sg-open in (X, m_X) . The complement of an m_X - ψ -closed set is called an m_X - ψ -open set.
- (c) an m_X - $\alpha\psi$ -closed [10] set if $m_X \text{-}\psi \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X - α -open in (X, m_X) . The complement of an m_X - $\alpha\psi$ -closed set is called an m_X - $\alpha\psi$ -open set.

III. PROPERTIES OF $\mathcal{M}_X\alpha\psi(\overline{\mathcal{H}})$

Definition 3.1. A function $f:(X, m_X) \rightarrow (Y, m_Y)$ is called

- (a) $\mathcal{M}_X\alpha\psi$ -continuous [10] if $f^{-1}(V)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) for every m_X -closed set V of (Y, m_Y) .
- (b) $\mathcal{M}_X\alpha\psi$ -irresolute if $f^{-1}(V)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) for every $\mathcal{M}_X\alpha\psi$ -closed set V of (Y, m_Y) .

Definition 3.2. A function $f:(X, m_X) \rightarrow (Y, m_Y)$ is called $\mathcal{M}_X\alpha\psi$ -homeomorphism (briefly $\mathcal{M}_X\alpha\psi(\overline{\mathcal{H}})$), if both f and f^{-1} are $\mathcal{M}_X\alpha\psi$ -irresolute and f is bijective.

Lemma 3.3. Let $B \subseteq H \subseteq (X, m_X)$ and let $m_X \setminus H$ be the relative m -spaces of H .

- (a) If B is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) , then B is $\mathcal{M}_X\alpha\psi$ -closed relative to H .
- (b) If B is $\mathcal{M}_X\alpha\psi$ -closed in a subspace $(H, m_X \setminus H)$ and if H is closed in (X, m_X) .

Note that a subset H of m -space (X, m_X) is $\mathcal{M}_X\alpha\psi$ -clopen if and only if H is open and $\mathcal{M}_X\alpha\psi$ -closed. We prepare

some notations. Let $f : X \rightarrow Y$ be a function and H a subset of (X, m_X) . Let $f \setminus H : H \rightarrow Y$ be the restriction of f to H . We define a function $r_{H,K(f)} : H \rightarrow K$ by $r_{H,K(f)}(x) = f(x)$ for any $x \in H$, where $K = f(H)$. Then, $f \setminus H = j \circ r_{H,K(f)}$ holds, where $j : K \rightarrow Y$ is an inclusion.

Theorem 3.4. Let H and K be subset of (X, m_X) and (Y, m_Y) respectively.

- (a) If $f:(X, m_X) \rightarrow (Y, m_Y)$ is $\mathcal{M}_X\alpha\psi$ -irresolute and if H is a $\mathcal{M}_X\alpha\psi$ -clopen subset of (X, m_X) , then the restriction $f \setminus H : (H, m_X \setminus H) \rightarrow (Y, m_Y)$ is $\mathcal{M}_X\alpha\psi$ -irresolute.
- (b) Suppose that K is a $\mathcal{M}_X\alpha\psi$ -clopen subset of (Y, m_Y) . A function $k : (X, m_X) \rightarrow (K, m_Y \setminus K)$ is $\mathcal{M}_X\alpha\psi$ -irresolute if and only if $j \circ k : (X, m_X) \rightarrow (Y, m_Y)$ is $\mathcal{M}_X\alpha\psi$ -irresolute, where $j : (K, m_Y \setminus K) \rightarrow (Y, m_Y)$ is an inclusion.
- (c) If $f:(X, m_X) \rightarrow (Y, m_Y)$ is a $\mathcal{M}_X\alpha\psi, \mathcal{H}$ such that $f(H) = K$ and K are $\mathcal{M}_X\alpha\psi$ -clopen subset, then $r_{H,K(f)} : (H, m_X \setminus H) \rightarrow (K, m_Y \setminus H)$ is also $\mathcal{M}_X\alpha\psi, \mathcal{H}$.

Proof: (a) Let F be a $\mathcal{M}_X\alpha\psi$ -closed set of (Y, m_Y) . Since f is $\mathcal{M}_X\alpha\psi$ -irresolute, $(f \setminus H)^{-1} = f^{-1} \cap H$, H is $\mathcal{M}_X\alpha\psi$ -closed, $(f \setminus H)^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in $(H, m_X \setminus H)$ by Lemma 3.3(a). Therefore $f \setminus H$ is $\mathcal{M}_X\alpha\psi$ -irresolute.

(b) *Necessity:* Let F be a $\mathcal{M}_X\alpha\psi$ -closed set of (Y, m_Y) . Then $(j \circ k)^{-1}(F) = k^{-1}(j^{-1}(F)) = k^{-1}(F \cap H)$ is a $\mathcal{M}_X\alpha\psi$ -closed in $(K, m_Y \setminus K)$ by Lemma 3.3(a). Therefore $j \circ k : (X, m_X) \rightarrow (Y, m_Y)$ is $\mathcal{M}_X\alpha\psi$ -irresolute.

Sufficiency: Let V be a $\mathcal{M}_X\alpha\psi$ -closed set of $(K, m_Y \setminus K)$. By Lemma 3.3(b), $(j \circ k)^{-1}(V) = k^{-1}(j^{-1}(V)) = k^{-1}(F \cap V) = k^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) . Therefore k is $\mathcal{M}_X\alpha\psi$ -irresolute.

(c) First, it suffices to prove $r_{H,K(f)} : (H, m_X \setminus H) \rightarrow (K, \sigma \setminus K)$ is $\mathcal{M}_X\alpha\psi$ -irresolute. Let F be a $\mathcal{M}_X\alpha\psi$ -closed subset of (Y, m_Y) .

$(j \circ r_{H,K(f)})^{-1}(F) = (f \setminus H)^{-1}(F) = (f \setminus H)^{-1}(F \cap K) = f^{-1}(F \cap K) = f^{-1}(F) \cap K$ is $\mathcal{M}_X\alpha\psi$ -closed in $(H, m_X \setminus H)$ and hence $j \circ r_{H,K(f)}$ is $\mathcal{M}_X\alpha\psi$ -irresolute. By (b) $r_{H,K(f)}$ is $\mathcal{M}_X\alpha\psi$ -irresolute.

Next we show that $r_{H,K(f)}^{-1} : (K, m_Y \setminus K) \rightarrow (H, m_X \setminus H)$ is $\mathcal{M}_X\alpha\psi$ -irresolute. Since $(r_{H,K(f)})^{-1} = r_{H,K(f^{-1})}$ and since f^{-1} is $\mathcal{M}_X\alpha\psi$ -irresolute, then using the first argument above for f^{-1} we have $(r_{H,K(f)})^{-1}$ is $\mathcal{M}_X\alpha\psi$ -irresolute. Therefore $r_{H,K(f)}$ is a $\mathcal{M}_X\alpha\psi, \mathcal{H}$. By using Theorem 3.4, for

a $\mathcal{M}_X\alpha\psi$ -clopen subset H of (X, m_X) , we have a homomorphism called restriction $(r_H)^\# : \mathcal{M}_X\alpha\psi, \mathcal{H}(X, H; m_X) \rightarrow \mathcal{M}_X\alpha\psi, \mathcal{H}(H, m_X \setminus H)$ as follows: $(r_H)^\#(f) = r_{H, K(f)}$ for any $f \in \mathcal{M}_X\alpha\psi, \mathcal{H}(X, H; m_X)$. To prove that $(r_H)^\#$ is onto we prepare the following:

Lemma 3.5. (PASTING LEMMA FOR $\mathcal{M}_X\alpha\psi$ -IRRESOLUTE FUNCTIONS)

Let (X, m_X) be a m -space such that $X = A \cup B$ where A and B are $\mathcal{M}_X\alpha\psi$ -clopen subsets. Let $f : (A, m_X \setminus A) \rightarrow (Y, m_Y)$ and $g : (B, m_X \setminus B) \rightarrow (Y, m_Y)$ be $\mathcal{M}_X\alpha\psi$ -irresolute functions such that $f(x) = g(x)$ for every $x \in A \cap B$. Then the combination $f \nabla g(x) = f(x)$ for any $x \in A$ and $f \nabla g(y) = g(y)$ for any $y \in B$.

Proof: Let F be $\mathcal{M}_X\alpha\psi$ -closed set of (Y, m_Y) . Then $(f \nabla g)^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, $f^{-1}(F) \in \mathcal{M}_X\alpha\psi\mathcal{C}(X, m_X)$ by using Lemma. It follows from Theorem 3.17 [13] that $f^{-1}(F) \cup g^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) . Therefore $(f \nabla g)^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) and hence $f \nabla g$ is $\mathcal{M}_X\alpha\psi$ -irresolute.

Theorem 3.6. If H is a $\mathcal{M}_X\alpha\psi$ -clopen subset of (X, m_X) , then $(r_H)^\# : \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X) \rightarrow \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, m_X \setminus H)$ is an onto homomorphism.

Proof. Let $k \in \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, m_X \setminus H)$. By Theorem 3.4, $J_1 \circ k : (H, m_X \setminus H) \rightarrow (X, m_X)$ is $\mathcal{M}_X\alpha\psi$ -irresolute, where $J_1 : (H, m_X \setminus H) \rightarrow (X, m_X)$ is an inclusion. Similarly it is shown that $J_2 \circ l_{X \setminus H} : (X \setminus H, m_X(X \setminus H)) \rightarrow (X, m_X)$ is $\mathcal{M}_X\alpha\psi$ -irresolute, where $J_2 : (X \setminus H, m_X(X \setminus H)) \rightarrow (X, m_X)$ is an inclusion.

By using Lemma 3.5, the combination $(J_1 \circ k) \nabla (J_2 \circ l_{X \setminus H}) : (X, m_X) \rightarrow (X, m_X)$, say k_1 is $\mathcal{M}_X\alpha\psi$ -irresolute. It is easily shown that $k_1(x) = k(x)$ for any $x \in H$ and k_1 is bijective and $k_1^{-1} = (J_1 \circ k^{-1}) \nabla (J_2 \circ l_{X \setminus H}) : (X, m_X) \rightarrow (X, m_X)$ is also $\mathcal{M}_X\alpha\psi$ -irresolute. Therefore $k_1 : (X, m_X) \rightarrow (X, m_X)$ is the required $\mathcal{M}_X\alpha\psi, \mathcal{H}$ and $(r_H)^\#(k_1) = k$ holds and hence $(r_H)^\#$ is onto. We define an equivalence relation R on $\mathcal{M}_X\alpha\psi, \mathcal{H}(X, H; m_X)$ as follows:

$f R h$ if and only if $f(x) = h(x)$ for any $x \in H$. Let $[f]$ be the equivalence class of f . Let $H = \{f \mid f \in \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X) \text{ and } f(x) = x \text{ for any } x \in H\}$. Then, $H = k \in r_{H, K}^\#$ and this is normal subgroup of

$\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X)$. The factor group of $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X)$ by H is $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X) \setminus H = \{fH \mid f \in \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(X, H; m_X)\}$, where $fH = \{\mu(f, k) \mid k \in H\} = [f]$.

Since $(r_H)^\#$ is onto by Theorem 3.6, then the relation between the groups $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X)$ is investigated as follows:

Theorem 3.7. If H is $\mathcal{M}_X\alpha\psi$ -clopen subset of (X, m_X) , then $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, m_X \setminus H)$ is isomorphic to the factor group $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X) \setminus H$.

Proof. By Theorem, $(r_H)^\# : \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X) \rightarrow \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, m_X \setminus H)$ is an onto homomorphism. Thus, we have the required isomorphism, $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, \tau \setminus H) \cong \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X) \setminus H$.

Theorem 3.8. If $\alpha : (X, m_X) \rightarrow (Y, m_Y)$ is a $\mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})$ such that $\alpha(H) = K$, then there is an isomorphism, $\alpha^\# : \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X) \rightarrow \mathcal{M}_X\alpha\psi(\tilde{\mathcal{H}})(H, H; m_X)$.

Proof. The isomorphism $\alpha^\#$ is defined by $\alpha^\#(f) = \alpha \circ f \circ \alpha^{-1}$. Let $(X \setminus R, m_X \setminus R)$ be the quotient m -space of (X, m_X) by an equivalence relation R on X and let $\pi : (X, m_X) \rightarrow (X \setminus R, m_X \setminus R)$ be the canonical projection.

Definition 3.9. A space (X, m_X) is called $\mathcal{M}_X\alpha\psi$ -connected if X cannot be expressed as the disjoint union of two non-empty $\mathcal{M}_X\alpha\psi$ -closed sets.

Definition 3.10. A function $f : (X, m_X) \rightarrow (Y, m_Y)$ is called $\mathcal{M}_X\alpha\psi$ -closed if $f(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (Y, m_Y) for every closed set F of (X, m_X) .

Definition 3.11. A space (X, m_X) is called $T^\# \mathcal{M}_X\alpha\psi$ -space if every $\mathcal{M}_X\alpha\psi$ -closed set is m_X -closed.

Theorem 3.12. Let F be a subset of $(X \setminus R, m_X \setminus R)$. If $\pi : (X, m_X) \rightarrow (X \setminus R, m_X \setminus R)$ is $\mathcal{M}_X\alpha\psi$ -closed function and $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) , then F is $\mathcal{M}_X\alpha\psi$ -closed, where (X, m_X) is a $T^\# \mathcal{M}_X\alpha\psi$ -space.

Proof. Let $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$ -closed in (X, m_X) . Then $\pi^{-1}(F)$ is closed in (X, m_X) , since (X, m_X) is $T^\# \mathcal{M}_X\alpha\psi$ -space. Then $\pi(\pi^{-1}(F)) = F$ is $\mathcal{M}_X\alpha\psi$ -closed in $(X \setminus R, m_X \setminus R)$, since π is a $\mathcal{M}_X\alpha\psi$ -closed map.

Theorem 3.13. If $\pi: (X, m_X) \rightarrow (X \setminus R, m_X \setminus R)$ is $\mathcal{M}_X \alpha \psi$ -continuous and the subset F is $\mathcal{M}_X \alpha \psi$ -closed $(X \setminus R, m_X \setminus R)$, then $\pi^{-1}(F)$ is $\mathcal{M}_X \alpha \psi$ -closed in (X, m_X) .

Proof. Let F be a closed set in $(X \setminus R, m_X \setminus R)$. Then F is $\mathcal{M}_X \alpha \psi$ -closed in $(X \setminus R, m_X \setminus R)$. Since π is $\mathcal{M}_X \alpha \psi$ -continuous, Then $\pi^{-1}(F)$ is $\mathcal{M}_X \alpha \psi$ -closed in (X, m_X) .

Theorem 3.14. If the bijective map $\pi: (X, m_X) \rightarrow (X \setminus R, m_X \setminus R)$ is $\mathcal{M}_X \alpha \psi$ -continuous and (X, m_X) is $\mathcal{M}_X \alpha \psi$ -connected, then $(X \setminus R, m_X \setminus R)$ is $\mathcal{M}_X \alpha \psi$ -connected.

Proof. Suppose that $(X \setminus R, m_X \setminus R)$ is not $\mathcal{M}_X \alpha \psi$ -connected. Therefore $X \setminus R = A \cup B$, where A and B are $\mathcal{M}_X \alpha \psi$ -closed set. Then $\pi^{-1}(A)$ and $\pi^{-1}(B)$ are $\mathcal{M}_X \alpha \psi$ -closed in (X, m_X) such that $X = \pi^{-1}(A) \cup \pi^{-1}(B)$. Therefore (X, m_X) is not $\mathcal{M}_X \alpha \psi$ -connected. It is a contradiction. Therefore $(X \setminus R, m_X \setminus R)$ is $\mathcal{M}_X \alpha \psi$ -connected.

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