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Review on Curvelet Transform and Its Applications

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Abstract **- The Curve let Transform gives better performance in terms of PSNR. Face recognition is very important for many applications such as: video surveillance, criminal investigations and forensic applications, secure electronic banking, mobile phones, credit cards, secure access to buildings. The curvelet transform is a multi scale directional trans-form, which allows an almost optimal non adaptive sparse representation of objects with edges. Curvelet have also proven useful in diverse fields beyond the traditional image processing application**

Keyword: **Image processing, Curve let transform, Wavelet Transform.**

I. Introduction

In the field of Image processing different types of method can be use for the analysis of image in different field such as De noising, Compression, Face recognition, Bio medical application etc. These analysis of image is based on the different types of transform is Fourier transform, Wavelet Transform, Curve let transform, Now-a-day Wavelet and Curvelet transform is used in all field of image processing.

A. Digital Image Processing System

A typical digital image processing system consists of image segmentation, feature extraction, pattern recognition, and thresholding and error classification. Image processing aims at extracting the necessary information from the image. The image needs to be reduced to certain defining characteristics and the analysis of these characteristics gives the relevant information shows a process flow diagram of a typical digital image processing system, showing the sequence of the operations.

B. Wavelet Transform

In the field of signal processing we can get time -Amplitude representation of signal but this is not properrepresentation of signals. For proper representation, we use frequency domain representation, but in the Fourier transform analysis time information is not obtained. To overcome this drawback of Fourier transform wavelet transform are used, Wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function (t) as a superposition of a set of such wavelets or basic functions. These basic functions or baby wavelets are obtained from a single prototype wavelet called the mother wavelet, by dilation and translation operation. Discrete Wavelet Transform of a finite length signal s(n) having N components, for image is expressed by an N x N matrix.

C. Curvelet Transform

To overcome the draw back of Wavelet Transform, curve let Transform is developed, Curve let Transform is very effective modal that not only consider a multi scale Time – Frequency local portion but also make use of the direction of features. It was developed by Candes and Donoho in 1999,there are two types of Curvelet transform is unequally spaced Fast Fourier Transform and wrapping based fast Curve let Transform, in curve let transform the width and length are related by the relation Width~ Length 2 that is known as parabolic or anisotropic scaling . Moreover, frame elements in curve let indexed by scale, location and orientation parameters in contrast to wavelets where elements have only scale and location. This transform can we used for both continuous and digital domain. In this angle polar wedges or angle trapezoid window is used in frequency domain .Initially construction of Curve let transform is redesigned as fast discrete Curve let transform (FDCT) by Candes in 2006.This is second generation curve let transform is meant to be simpler to understand and use . DCT can be implemented by wrapping based fast discrete curve let at a given scale. At a given scale and orientation both the image and the curve let are transformed into Fourier domain. The product of curve let and the image are obtained in the fourier domain. Inverse FFT is applied to the above product to obtain a set of curve let coefficients. In order to perform IFT the trapezoidal wedge thus obtained from the frequency response of a curve let is wrapped into a rectangular support. The spectrum inside the wedge is tilted periodically. Thus the rectangular region collects the wedge's fragmented portions by periodic tilting.

II. Importance of Curvelets Over Wavelets

Curvelets will be superior over wavelets in following cases:

i) This transform is optimally sparse representation of Objects with edges.

ii) This transform is optimal image reconstruction in severely ill-posed problems.

ii) This transform is optimal sparse representation of wave propagators.[3]

A.Continuous Curvelet Transform

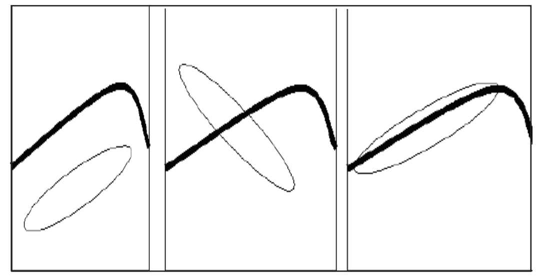
The Continuous Curvelet Transform has gone through twomajor revisions. The first Continuous Curvelet Transform (commonly referred to as the "Curvelet '99" transform now) used a complex series of steps involving the ridgelet analysis of the radon transform of an image. Performance was exceedingly slow. The algorithm was modified in 2003. The use of the Ridgelet Transform was discarded, thus reducing the amount of redundancy in the transform and increasing the speed considerably.

B. In A Heuristic Argument Is Made That All Curvelets Fall Into One Of Three Categories.

i) A curvelet whose length-wise support does not intersect a discontinuity. The curvelet coefficient magnitude will be zero. (Fig. 1.a)

ii) A curvelet whose length-wise support intersects with a discontinuity, but not at its critical angle. The curvelet coefficient magnitude will be close to zero. (Fig. 1.b)

iii) A curvelet whose length-wise support intersects with a discontinuity, and is tangent to that discontinuity. The curvelet coefficient magnitude will be much larger than zero. (Fig. 1.c) [2]



A B C

Fig.1 Curvelet Type A, Curvelet Type B and Curvelet Type C

III. Description of Curvelet Transform

Basically, curvelet transform extends the ridgelet transform to multiple scale analysis. Therefore, let's start from the definition of ridgelet transform. Given an image function f(x, y), the continuous ridgelet transform is given as

$$\Re_f(a,b,\theta)=\iint \psi_{a,b,\theta}(x,y)f(x,y)dxdy$$

Where a>0 is the scale, bθR is the translation and θ [0,2Π] is the orientation. The ridgelet is defined as:

$$\psi_{a,b,\theta}(x,y)=a^{-\frac{1}{2}}\psi(\frac{x\cos\theta+y\sin\theta-b}{a})$$

Fig. 2 shows a typical ridgelet [11]. It is oriented at an angle θ, and is constant along lines: x•cosθ+ y•sinθ= const. It can be seen that a ridgelet is linear in edge direction and is much sharper than a conventional sinusoid wavelet.

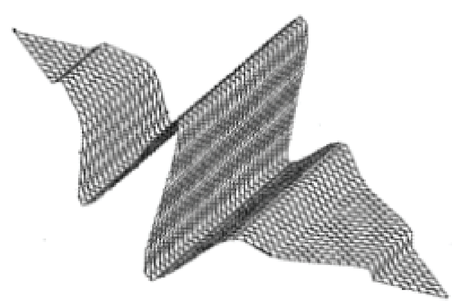


Fig.2 A ridgelet waveform.

For comparison, the 2-D wavelet is given as:

$$\psi_{a_1,a_2,b_1,b_2}(x,y) = a_1^{-\frac{1}{2}} a_2^{-\frac{1}{2}} \psi\left(\frac{x-b_1}{a_1}\right)\psi\left(\frac{y-b_2}{a_2}\right)$$

As can be seen, the ridgelet is similar to the 2-D wavelet except that the point parameters (b1, b2) are replaced by the line parameters (b,θ. In other words, the two transforms are related by:

Wavelet : ψ scale,point-position

Ridgelet : ψ scale,line-position

This means that ridgelet can be tuned to different orientations and different scales to create the curvelets,. Fig. 3 shows a single curvelet and the curvelets tuned to two scales and different number of orientations at each scale , curvelets have a complete cover of the spectrum in frequency domain. That means, there is no loss of information in curvelet transform in terms of capturing the frequency information from images. Fig. 4 shows the curvelet tiling and cover of the spectrum of a 512x512 images with 5 scales [4]. The shaded wedge shows the frequency response of a curvelet at orientation 4 and scale 4. It can be seen, the spectrum cover by curvelets is complete. In contrast, there are many holes in the frequency plan of Gabor filters (Fig. 3 bottom).

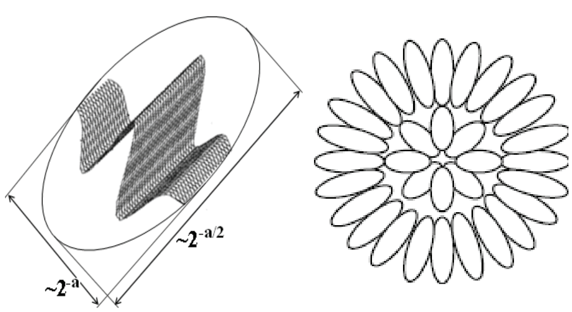


Fig. 3 A single curvelet. & curvelets tuned to different scales and orientations.

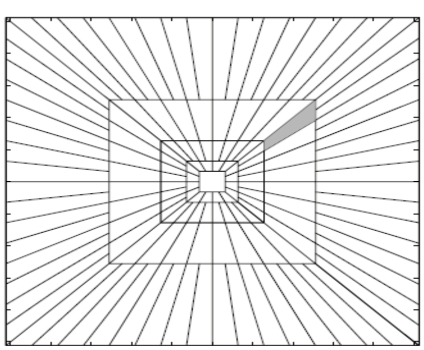


Fig. 4 The tiling of frequency plan by curvelets [9]

IV. Application of Curvelet Transform

In this section, we shall review applications of the curve lets in image processing, fluid mechanics, seismic exploration, solving of PDEs, and compressed sensing, to show their potential as an alternative to wavelet transforms in some scenarios.

A. Image Processing

In 2002, the first-generation curvelet transform was applied for the first time to image denoising by Starck et al. [7], and by Candµes and Guo [8]. The applications of the first-generation curvelets were extended to image contrast enhancement [11] and astronomical image representation in 2003, and to fusion of satellite images in 2005. After the effective second-generation curvelet transform [12] had been proposed in 2004, the applications of curvelets increased very fast in many fields involving image/video presentation, denoising and classification. In the firrst model [13], a curvelet shrinkage is applied to the noisy data, and the result is further processed by a projected total variation diffusion in order suppress pseudo-Gibbs artifacts. In the second model [14] The curvelet-based methods preserve the edges and textures well.

B. Compressed Sensing

In this we mention a new of applications of the curvelet transform is called compressed sensing or compressive sampling (CS), an inverse problem with highly incomplete measurements. CS [6] is a novel sampling paradigm, which carries imaging and compression simultaneously. The CS theory says that a compressible unknown signal can be recovered by a small number of random measurements using scarcity -promoting nonlinear recovery algorithms. The number of necessary measurements is considerably smaller than the number of needed traditional measurements that satisfy the Shannon/Nyquist sampling theorem, where the sampling rate has to be at least twice as large as the maximum frequency of the signal. The CS based data acquisition depends on its sparsity rather than its bandwidth. CS might have an important impact for designing of measurement devices in various engineering fields such as medical magnetic resonance (MRI) imaging and remote sensing, especially for cases involving incomplete and inaccurate measurements limited by physical constraints, or very expensive data acquisition.

C. Turbulence Analysis In Fluid Mechanics

The curvelets have been applied to study the non-local geometry of eddy structures and the extraction of the coherent vortex field in turbulent fows. Curvelets start to influence the field of turbulence analysis and have the potential to upstage the wavelet representation of turbulent flows addressed in [6]. The multi scale geometric property, implemented by means of curvelets, provides the framework for studying the evolution of the structures associated to the main ranges of scales defined in Fourier space, while keeping the localization in physical space that enables a geometrical study of such structures.

V. Conclusion

This paper present a review of Curvelet transform comprehensive insight into the different applications where it is being applied to. With more vehement research and expansion of scope, Curvelets can be applied to diverse fields so that revolutionary results can be obtained. This shows that the Curvelet Transforms are more suitable for the image data to represent the singularities over geometric structures in the image, than the Wavelet counterpart. Curvelet is designed to age data to represent handle the singularities on curves . Wavelets are effective for point singularities.

VI. Future Scope

1) The computational cost of curvelets is higher than that of wavelets. Software that would be able to implement fast curvelet algorithm that reduce the computational time have continued to be a burgeoning area of research. The application of Curvelets in 3D has also become a promising area of research.

2) Currently, the curvelets are constructed in Fourier domain. There is no explicit space-domain formulation for curvelets. This brings troubles in many applications such as numerical modeling of PDEs. How to build a space-domain formulation of curvelets remains a challenge.

3) Rotation and scale invariance will be investigated to further improve curvelet retrieval performance. Application of curvelet feature in color image retrieval and semantic learning will also be investigated.

4) How to explore suitable thresholding functions that incorporate and exploit the special characteristics of the curvelet transform? This issue is very important for curvelet applications involving edge detection, denoising, and numerical simulation.

5) Curvelet tansform can we use for Criminal investigation and forensic application in this the image recognition from all angle of body and motion of body also becomes area of research.

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