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Quantile Regression Estimation of Stock Market Volatility and its Causes

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Abstract - **Stock market volatility is the amount of uncertainty or risk about the size of changes in stock market security value. In this study, GARCH model was built to generate stock price volatility and quantile regression estimation was used to determine the cause of volatility in stock market at different quantile level. The study provides the graphical presentation of the coefficients estimated and the variables employed. The results of the study showed that the previous residuals (ARCH effect) are significantly contributed to stock market volatility at lower quantile level (0.1, 0.25, and 0.5) and the previous volatility significant only at higher quantile level (0.9), while only exchange rate return is significant among the external causes considered.**
Keywords: **GARCH Model, Quantile Regression, Stock Market, Volatility, Return.**

I. INTRODUCTION

Quantiles seem inseparably linked to the operations of ordering and sorting the sample observations that are usually used to define them. So it comes as a mild surprise to observe that we can define the quantiles through a simple alternative expedient as an optimization problem. Just as we can define the sample mean as the solution to the problem of minimizing a sum of squared residuals, we can define the median as the solution to the problem of minimizing a sum of absolute residuals. The symmetry of the piecewise linear absolute value function implies that the minimization of the sum of absolute residuals must equate the number of positive and negative residuals, thus assuring that there are the same number of observations above and below the median.

Financial regulations usually require banks reports their daily risk measures called value at risk (VaR). VaR model are the most commonly used measure of market risk in the financial industry (Lauridsen 2000). Let Y be the financial return, so that y satisfying $P(Y \leq y) = p$ for a given low value of p is the VaR. the variable Y may depend on covariates x such as exchange rates. Clearly, VaR estimation relates to extreme quantile estimation through estimating the tail of financial return. The distribution of financial return could also be illustrated by several quantiles. The common approach to estimating the distribution of one period return in financial models is to forecast the volatility and then to make a Gaussian assumption (see Hall and White (1998)). Market returns, however, are frequently found to have more kurtosis than a normal distribution. A general discussion of using quantile regression for return- based analysis was given by Bassett and Chen (2001). Besides, several authors have employing quantile regression for generalized autoregressive conditional heteroskedasticity (GARCH) in various fields.

There are some studies on estimation and applications of estimating conditional quantiles. In particular, based on a relation between expectile and quantile, Taylor (2008a) and Kuan et al (2009) estimate conditional quantiles using asymmetric least squares methods. Taylor (2008b) proposes the exponentially weighted quantile regression for estimating time-varying quantiles. Gourieroux and Jasiak (2008) proposed the Dynamic Additive Quantile (DAQ) model for calculating conditional quantiles. Giot and Laurent (2003) model Value-at-Risk using parametric ARCH models based on skewed *t*-distributions. Coroneo and Veredas (2008) analyze the density of high frequency financial returns using quantile regression of ARCH models and conducted intradaily Value-at-Risk assessment.

Zhijie and Roger (2009), study estimation of conditional quantiles for GARCH models using quantile regression. They propose a simple and effective two-step approach of quantile regression estimation for linear GARCH time series. In the first step, a quantile autoregression sieve approximation for the GARCH model by combining information was employed over different quantiles; second stage estimation for the GARCH model is then carried out based on the first stage minimum distance estimation of the scale process of the time series. Asymptotic properties of the sieve approximation, the minimum distance estimators, and the final quantile regression estimators employing generated regressors are studied. These results are of independent interest and have applications in other quantile regression settings. The Monte Carlo and empirical application results indicate that the proposed estimation methods outperform some existing conditional quantile estimation methods.

Sangyeol and Jungsik (2012) study the quantile regression estimator for GARCH models. They formulated the quantile regression problem by a reparametrization method and verify that the obtained quantile regression estimator is strongly consistent and asymptotically normal under certain regularity conditions

Serpil and Mesut (2013) employed a BEKK-MGARCH model approach to generate the conditional variances of monthly stock exchange prices, exchange rates and interest rates for Turkey before the effects of global economic crisis hit Turkey, the results indicate a significant transmission of shocks and volatility among these three financial sectors.

Also Beum-Jo (2014) investigated the systematic impact of the European Monetary System (EMS) on asymmetry in volatility of exchange rates. It seems plausible that the symmetric fluctuation band in the EMS affects asymmetric volatility and this is dominant at extreme returns. Examine the plausibility; they proposed quantile regression for threshold GARCH models (QRTGARCH), which allows an asymmetric reaction of conditional volatility to shocks without any rigid distributional assumptions. Further, it is well suited to precisely capture the asymmetric behaviors of conditional volatility over different levels of returns. The empirical finding suggests that the EMS seems to have some systematic effect on the asymmetry in volatility at moderate level of unpredictable returns. Especially, the estimation results of the QRTGARCH show that after the EMS conditional volatility for most of EMS currencies tends to grow more significantly in reaction to positive shock than negative shock at 0.1 quantile of returns distribution, so that as the unpredictable returns go down, the systematic effect of the EMS on asymmetry in volatility becomes more significant.

Ilouba (2009) who investigated the relationships among stock prices, oil prices and microeconomic variables in Nigeria failed to capture volatility at different point of variation instead they used the exponential generalized autoregressive conditional Heteroskedsticity (EGARCH) estimation as measure of volatility. This study will differ from existing studies in Nigeria and beyond by using estimates obtained from quantile regression of generalized autoregressive conditional Heteroskedsticity estimations of variables of interest to investigate the relationship between stock prices and its causes at different quantiles level (0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95).

II. MATERIALS AND METHODS

This section discusses the sample selection procedure, variables selection, the model used for the research and the statistical techniques.

The study employed Ilouba (2009) data set, measured on quarterly basis between 1981 and 2008. It contains stock prices extracted from Nigeria stock exchange bulletin, exchange rates interest rates and inflation rate from Central Bank of Nigeria annual statistical bulletin. The data series will be subjected to the appropriate time series. In the estimations process, we will introduce estimation technique quantile regression and then apply it to build standard deviation of the GARCH model of stock price return and the external cause of the volatility i.e exchange rates, interest rates and inflation rates returns. Quantile regression GARCH estimation will provide the actual risk of the stock market return and previous quarters return information about volatility at various quantile points of the distribution.

A.Quantile Regression Estimation of GARCH Models

Following Bollerslev (1986) and Taylor (1986), the stock market volatility estimates will be obtained using Quantile Regression for Generalized Autoregressive Conditional Heteroskedascity (QRGARCH) model developed by Zhijie and Roger (2009). Therefore, QRGARCH *(p,q)* model were specified as follow:

In the original linear form of the GARCH model, u_t follows a GARCH *(p,q)* process if

$$u_t = \sigma_t . \varepsilon_t, \qquad (1)$$

$$\sigma_t = \beta_0 + \beta_1 \sigma_{t-1} + \ldots + \beta_p \sigma_{t-p} + \gamma_1 |u_{t-1}| + \ldots + \gamma_q |u_{t-q}| \quad (2)$$

$$\sigma_t = \beta_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-p} + \sum_{j=1}^{q} \gamma_j u_{t-q}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2 + \gamma_1 u_{t-1}^2 + \ldots + \gamma_q u_{t-q}^2 \quad (3)$$

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-p}^2 + \sum_{j=1}^{q} \gamma_j u_{t-q}^2$$

where $\beta_0 > 0, (\gamma_1, \ldots, \gamma_q)^T \in \Re_+^q$, and ε_t are independent and identically distributed with mean zero and unknown distribution function $F_\varepsilon(.)$, to estimate $\tau - th$ conditional quantile of u_t and conditional volatility as well as GARCH parameters. Let F_{t-1} represents in formation up to time $t-1$, the $\tau - th$ conditional quantile of u_t is given by

$$Q_{ut}(\tau | F_{t-1}) = \theta(\tau)^T z_t, \qquad (4)$$

where

$$z_t = (1, \sigma_{t-1}, \ldots, \sigma_{t-p} |u_{t-1}|, \ldots, |u_{t-q}|)^T,$$

$$\theta(\tau)^T = (\beta_0, \beta_1, \ldots, \beta_p, \gamma_1, \ldots, \gamma_q) F^{-1}(\tau).$$

Notice that $\sigma_{t-j} F^{-1}(\tau) = Q_{ut-j}(\tau | F_{t-j-1})$, the conditional quantile $Q_{ut}(\tau | F_{t-1})$ has the following CAViaR*(p,q)* representation

$$Q_{ut}(\tau | F_{t-1}) = \beta_0^* + \sum_{i=1}^{p} \beta_i^* Q_{ut-i}(\tau | F_{t-i-1}) + \sum_{j=1}^{q} \gamma_j^* |u_{t-j}| \quad (5)$$

where

$\beta_0^* = \beta_0(\tau) = \beta_0 F^{-1}(\tau), \beta_i^* = \beta_i, i = 1,\ldots,p$ and $\gamma_j^* = \gamma_j(\tau) = \gamma_j F^{-1}(\tau), j = 1,\ldots,q.$

More generally, a time series y_t in a regression model,

$$y_t = \mu^T X_t + u_t, \qquad (6)$$

where the residuals u_t follow a linear GARCH process as characterized by (3.1), the $\tau - th$ conditional quantile of y_t in model (3.6) is given by

$$Q_{yt}(\tau | F_{t-1}) = \mu^T X_t + \theta(\tau)^T z_t, \qquad (7)$$

where $X_t = (1, x_{2t}, \ldots, x_{kt})^T$. In the above problem, the key component is the estimation of conditional quantiles of process u_t: $Q_{ut}(\tau | F_{t-1}) = \theta(\tau)^T z_t$.

Since z_t contains $\sigma_{t-k}(k = 1,\ldots,q)$ which in turn depend on unknown parameters $\theta = (\beta_0, \beta_1, \ldots, \beta_p, \gamma_1, \ldots, \gamma_q)$, z_t is written as $z_t(\theta)$ whenever it is necessary to emphasize the nonlinearity and its dependence on θ. To estimate the conditional quantiles of the process u_t we consider the following nonlinear quantile regression estimator solving:

$$\min_\theta \sum_t \rho_\tau (u_t - \theta^T z_t(\theta)), \qquad (8)$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$. However, estimation of (8) for a fixed τ in isolation cannot yield a consistent estimate of θ since it ignores the global dependence of the σ_{t-k}'s on the entire function θ (·). If the dependence structure of u_t is characterized by (1) and (2)

Giving the GARCH model (3.1) and (2), let $A(L) = 1 - \beta_1 L - \cdots - \beta_p L^p$, and $B(L) = \gamma_1 + \cdots + \gamma_q L^{q-1}$, under regularity assumption the invertibility of $A(L)$, we obtain an ARCH(∞) representation for σ_t:

$$\sigma_t = \alpha_0 + \sum_{j=1}^{\infty} \alpha_j |u_{t-j}|, \qquad (9)$$

where the coefficients α_j satisfy summability conditions implied by the regularity conditions. For identification, we normalize α_0 = 1. Substituting the above ARCH (∞) representation into (1) and (2), we have

$$u_t = \left(\alpha_0 + \sum_{j=1}^{\infty} \alpha_j |u_{t-j}| \right) \varepsilon_t, \qquad (10)$$

and

$$Q_{u_t}(\tau | F_{t-1}) = \alpha_0(\tau) + \sum_{j=1}^{\infty} \alpha_j(\tau) |u_{t-1}|,$$

where $\alpha_j(\tau) = Q_{\varepsilon_t}(\tau)$, $j = 0, 1, 2, \cdots$

B.Variables Return

First we need to calculate the continuously compounded return of each period. This is calculated as:

$$R_n = \log(y_n) - \log(y_{n-1}) \quad (11)$$

where y_n = closing price, y_{n-1} = previous day closing price

Here we will analyze the data by assume that stock price return is the function of exchange rate, inflation rate and interest rate return in:

Linear regression model of the form:

$$SPR_t = \beta_0 + \beta_1 EXRR_t + \beta_2 INFRR_t + \beta_3 INTRR_t + u_t \quad (12)$$

where $\beta's$ are the parameters to be estimated , SPR_t is the stock price return, $EXRR_t$ is the exchange rate return, $INFRR_t$ is the inflation rate return, $INTRR_t$ is the interest rate return and the u_t is the residual. the purpose is to estimate the τ-th conditional quantile of u_t, (1) but we also provide robust estimators for the conditional volatility as well as the GARCH parameters (2).

Linear quantile regression for GARCH model of the form

$$\sigma_t(\tau) = \omega_\tau + \alpha_\tau |u_{t-1}| + \beta_\tau \sigma_{t-1} + \beta_{\tau,1} EXRR_t + \beta_{\tau,2} INFRR_t + \beta_{\tau,3} INTRR_t + \varepsilon_t(\tau) \quad (13)$$

Where τ is the conditional quantile function, σ_{t-1} is the lagged conditional standard deviation of stock price return, u_{t-1} is the lagged absolute stock price return, $EXRR_t$, $INFRR_t$ and $INTRR_t$ are the external regressor. The estimated parameters ω, α and β must satisfy non-negativity of the conditional standard deviation, i.e $0 < \omega, 0 \le \alpha, 0 \le \beta$ which means $\alpha + \beta < 1$.See Bollerslev (1986) and Nelson and Cao (1992) for details on the non-negativity and stationarity conditions of the GARCH process.

C. Unit Root Test

To determine the stationarity of the data, Augmented Dickey–Fuller test (ADF), and the Phillips–Perron test (PP) are used to examine the stationarity of the data (Dickey and Fuller 1981; Phillips and Perron 1988).

i. Augmented Dickey–Fuller test (ADF): The ADF test here consists of estimating the following regression (Gujarati and porter 2009):

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + \varepsilon_t \qquad (14)$$

where Δ is the first difference operator, t is the time trend, k denotes the number of lags used, and ε_t is the error term. β_i, δ and α_i are parameters. The null hypothesis that series Y_t is non-stationary can be rejected if δ is statistically significant with negative sign (Huarng et al. 2006). In addition, m shows the optimal lag order, which is chosen carefully using the Schwarz criterion (AIC) in empirical method.

ii. The Phillips–Perron test (PP): this is a complementary feature of the ADF unit root test Phillips and Perron use nonparametric statistical methods to take care of the serial correlation in the error terms without adding lagged difference terms. Since the asymptotic distribution of the PP test is the same as the ADF test statistic (Gujarati and Porter, 2009).

III. ANALYSIS AND RESULTS

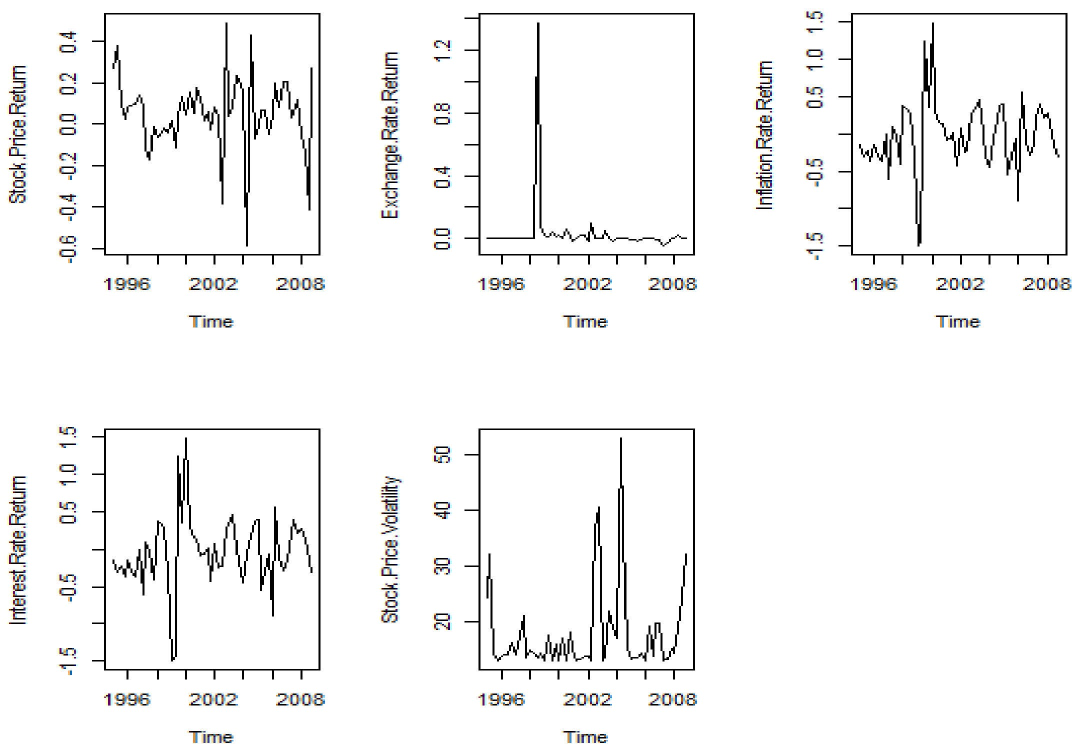

Fig.1 Time Series Plot of the stock price return, exchange rate return, inflation rate return, interest rate return and volatility of the stock price

TABLE 1a AUGMENTED DIKEY-FULLER (ADF) STATIONARITY (UNIT ROOT) TEST

Variables	**Augmented Dikey-Fuller (ADF)**		
	T-Statistics	P-value	Order of integration
SPR	-7.7335	0.0000	Stationary at level.
EXRR	-7.0570	0.0000	Stationary at level.
INFRR	-5.8103	0.0000	Stationary at level.
INTRR	-5.8310	0.0000	Stationary at level.

TABLE 1b PHILIPS-PERRON (PP) STATIONARITY (UNIT ROOT) TEST

Variables	**Philips-Perron (PP)**		
	T-Statistics	P-value	Order of integration
SPR	-7.7335	0.0000	Stationary at level.
EXRR	-7.0570	0.0000	Stationary at level.
INFRR	-5.8103	0.0000	Stationary at level.
INTRR	-5.8310	0.0000	Stationary at level.

TABLE 2 MEAN AND STANDARD ESTIMATION

mu	omega	alpha1	beta1
2.735 (2.1620) 0.2058*	166.7 (0.5198) 0.0013*	0.7021 (0.3063) 0.0219*	1.000e-08 - -

Source: From Author Analysis

Values without bracket and asteric are estimated parameters for internal cause of stock price volatility, with bracket and asteric are standard error and p-value respectively.

$$\mu = 2.7355$$

$$\sigma_t = 166.7 + 0.7021 u_{t-1} + 0.00000001 \sigma_{t-1}$$

Above model is fitted GARCH which is the volatility or standard deviation of the stock price return. the model reveal that the previous quarterly information contribute 70% to the volatitily of the stock price while previous volatility cause little or noting to the volatility of the stock price return.

To determine the relationship between the stock price volatility and its determining factors at different quantile level we regress the stock price volatility extracted from GARCH model on previous qurterly volatility of stock price , previous quarterly returns of stock price, exchange rate return, inflation rate return and interest rate return and the result is as follow:

TABLE 3 QANTILE REGRESSION PARAMETERS ESTIMATION OF STOCK PRICE VOLATILITY

Quantile	ω	u_{t-1}	σ_{t-1}	$EXRR$	$INFRR$	$INTRR$
0.1	2.00620 (0.3249) 0.0000*	19.75565 (2.6612) 0.0000*	0.00441 (0.0133) 0.7409*	1.20915 (0.2071) 0.0000*	0.00003 (0.1559) 0.9998*	0.60127 (2.5491) 0.8145*
0.25	12.2589 (0.94526) 0.00000*	16.2232 (0.74928) 0.00000*	0.0101 (0.06897) 0.88431*	0.9420 (0.10563) 0.00000*	0.1169 (0.13651) 0.39621*	1.0971 (0.9925) 0.27449*
0.50	1.70608 (2.9277) 0.0002*	14.87129 (6.1991) 0.0204*	0.14495 (0.2058) 0.4846*	0.07292 (0.8670) 0.9333*	0.62997 (1.3701) 0.6477*`	9.65276 (9.1664) 0.2976*
0.75	10.6282 (6.4211) 0.1044*	-1.3505 (17.3154) 0.9382*	0.5013 (0.4556) 0.2767*	-3.3793 (2.5434) 0.1903*	-0.3622 (4.0042) 0.9283*	22.2123 (17.9633) 0.2223*
0.90	8.82412 (3.6210) 0.0186*	-29.2631 (16.6347) 0.0849*	1.22788 (0.2855) 0.0001*	-9.48311 (2.2449) 0.0001*	0.78838 (3.4120) 0.8182*	-23.5234 (64.5563) 0.7172*

Source: From Author Analysis

Values without bracket and ateric are estimated parameters for internal and external cause of stock price volatility, with bracket and asteric are standard error and p-value respectively.

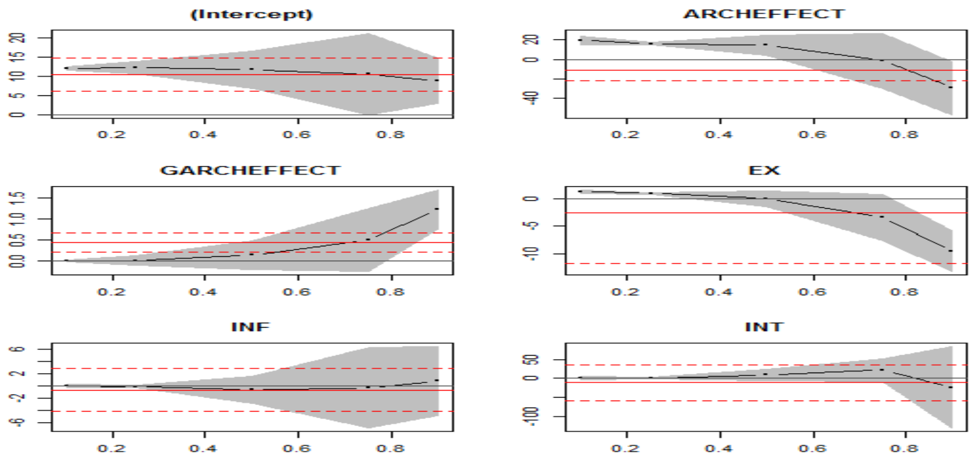

Fig.2 Graphical Representation of the coefficient of the variables

IV. DISCUSSION

From the analysis, it can be observed that all the variables are stationary at level and table 2 reveal the mean and standard deviation of stock price, such that the previous quarterly stock price returns contributed 70% to the volatility of the stock price while the previous quarterly volatility of stock price has little or no contribution to the volatility. Table 3 show the quantile estimation of stock price volatility. It can be found that the previous quarterly residuals (u_{t-1}) is significant at low quantile level (0.1, 0.25 and 0.50) with coefficients 19.76, 16.22 and 14.87 respectively, meaning that there is ARCH effect on the stock price volatility at the lower quntile but there is no ARCH effect at 0.75 and 0.9 quantiles. Also the previous quarterly volatility of the stock price (σ_{t-1}) found to be significant with at higher quantile level, meaning that there is GARCH effect at 0.9 quantile level only. The external caurse exchange rate return found to be significant at 0.1, 0.25 and 0.9 quantile level while the inflation and interest rate return does not significant at any of the quantile level, meaning that only exchange rate has contribution to stock price volatility.

V. CONCLUSION

This study estimates the stock market volatility at different point of quantile levels, such that the volatility (standard deviation) of stock price is regress on previous quarter residuals, previous quarter volatility, exchange rate return, inflation rate return and intrest rate return. It can be found out from the results that the previous residuals of stock price cause volatility at lower quantile level and previous volatility of stock price cause volatility at higher quantile level while the only exchange rate return cause volatility among the external cause considered.

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