Asian Journal of Science and Applied Technology
ISSN: 2249-0698 Vol. 7, No.1, 2018, pp.17-19

Entanglement Measure Based on Matrix Realignment

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Abstract **- Quantum information processing is the essential requirement of quantum technology. The foundation of information processing is the quantum entanglement. We propose a new entanglement measure to quantify entanglement and it is based on matrix realignment technique. A comparative study of proposed measure with concurrence for different types of entangled states is also discussed.**
Keywords: **Concurrence, Density matrix, Quantum Information Processing, Quantum Entanglement, Matrix Realignment Criterion**

I. INTRODUCTION

Today's era is the era of information. From ancient time to today, humans are developing new technologies for smooth and comfortable life. For a successful civilization, good communication is required. Revolution of human communication may be considered from the origin of speech and from then till now we have designed smart phones, internet, banking and many more for faster and better communication. To ensure security of information, such as authenticity, data confidentiality and data integrity, cryptography technique is used. From the very first known technological invention, wheel to space craft, people have been inventing new ways to travel faster from one place to another but all modes of transportation require some physical distance to be covered. A new way of transportation is developed by combining the properties of telecommunication and transportation known as teleportation.

Cryptography, teleportation and many other phenomena require information to be processed and broadly known as quantum information processing. In information processing, as in physics, the classical view provides an incomplete approximation to an underlying quantum reality. Information processing which applies principles of quantum effects like interference and entanglement is called quantum information processing [1].In this field, laws of quantum physics and information theory bring together to perform tasks which are not possible within the framework of classical physics. It helps in speeding up of computation and achieving faster and even more secure communication. It is a multidisciplinary field which holds quantum physics, chemistry, machine learning, optics, simulation, and metrology, computer science and engineering as well as condensed matter physics, biochemistry, physical chemistry, inorganic and organic chemistry, and spectroscopy. The 1990's saw the development of a quantum theory of information, based on the realization that entanglement can actually be exploited as a non-classical communication channel to perform information-processing tasks that would be impossible in a classical world. In a two-part commentary on the EPR paper [2], Schrödinger [3] identified entanglement as '*the* characteristic trait of quantum theory, the one that enforces its entire departure from classical lines of thought.' This hasled to an explosive surge of research among physicists and computer scientists on the application of information-theoretic ideas to quantum computation which exploits entanglement in the design of a quantum computer, so as to enable the efficient performance of certain computational tasks, to quantum communication such as quantum teleportation and to quantum cryptography that is guaranteed to be unconditionally secure against eavesdropping, by the laws of quantum mechanics. The key of quantum information processing is quantum entanglement. It is a phenomenon which has no classical analogue. Entangled states are basically superposed states. It was first pointed out by Einstein, Podolsky and Rosen (EPR) as incompleteness of quantum theory but according to Schrodinger it is the beauty of quantum mechanics.

Later on, after the successful experimental proof by Bell in 1964, EPR accepted that these states are not showing the incompleteness of quantum mechanics [4]. These states are sometimes also known as EPR pair or Bell states for two particles. By definition, entangled states are those which are not separable or cannot be written as product of individual particle's state. It is a type of correlation between particles irrespective of the physical distance separating them. Mathematical form of bipartite entangled states is:

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

A class of noisy entangled states termed as Werner state [5, 6] has the form

$$\hat{\rho} = p|\psi\rangle\langle\psi| + (1-p)\frac{I}{4}$$

here $|\psi\rangle$ is pure entangled state while $(1-p)\frac{I}{4}$ is the noise and p is a parameter predicting the pure entanglement present in it and varies in between 0 and 1. When p=0, the state is separable and p=1 corresponds maximally entangled state.

II. LITERATURE REVIEW

The application of entanglement mainly quantum information processing requires the states to be maximally entangled. To quantify and to detect entanglement several methods have been proposed by researchers [7]. Positive partial transpose criterion derived by Asher Peres [8] is well established criterion for detection of entanglement. This is necessary and sufficient condition for separability for 2⊗2 and 2⊗3 [7] systems but for other it is only sufficient not necessary. For a set of mixed bipartite state, if it's partially transposed matrix with respect to second particle, with elements

$$\langle n_1 n_2|\hat{\rho}^{pt}|m_1 m_2\rangle \equiv \langle n_1 m_2|\hat{\rho}|m_1 n_2\rangle$$

is a density operator then ρ̂ will be separable. There are some states which are entangled but cannot be detected through this criterion. Another method of detection was suggested by Miranowicz *et al*., in 2009 and Horodecki *et al.*, in 2007 [7, 9]. It is also based on manipulation of matrices like partial transposition criterion. It detects pure entanglement as well as PPT entanglement [8]. Realignment of matrix is defined as

$$\langle n_1 n_2|\hat{\rho}^{R}|m_1 m_2\rangle \equiv \langle n_1 m_1|\hat{\rho}|n_2 m_2\rangle \quad (1)$$

If the trace norm of realigned matrix $\hat{\rho}^R$ is not greater than one then the state is said to be separable otherwise entangled i.e. for entangled state $\|\hat{\rho}^R\| > 1$. The trace norm is defined as $\|A\| = Tr\left(\sqrt{A^{\dagger}A}\right)$. Any state that violates the realignment criterion there is a local uncertainty relation is violated. For Werner state same condition for entanglement is obtained as obtained by PPT criterion which is, for all the state is entangled, $p > \frac{1}{3}$. Let's take $|\phi^-\rangle$ as a part of Werner state and using (1), the realigned matrix will be

$$\hat{\rho}^R = \frac{1}{4}\begin{bmatrix} 1+p & 0 & 0 & 1-p \\ 0 & -2p & 0 & 0 \\ 0 & 0 & -2p & 0 \\ 1-p & 0 & 0 & 1+p \end{bmatrix} \text{ and}$$

$$\|\hat{\rho}^R\| = \frac{1+3p}{2} \quad (2)$$

From (2), it is observed that $\|\hat{\rho}^R\| > 1$ if $p > \frac{1}{3}$.For Bell state, $\|\hat{\rho}^R\| = 2$ i.e. maximum value of trace norm of realigned matrix is 2.

Negativity is an entanglement measure that attempts to quantify the negativity in the spectrum of the partially transposed density matrix [10]. For a separable state, the partial transposed density matrix will have non-negative eigenvalues. It is sufficient condition for $2\otimes 2$ and $2\otimes 3$ systems. It is defined as

$$N(\hat{\rho}) = \frac{\|\hat{\rho}^{pt}\| - 1}{2}$$

We have another measure for entanglement termed as concurrence. It is well defined quantitative measure of entanglement for bipartite system as well as for multipartite system. It is defined separately for pure and mixed state. For a pure state of a pair of qubits, the concurrence C(ψ) is defined as [11]

$$C(\psi) = \left|\langle\psi|\tilde{\psi}\rangle\right| = \sqrt{2\left[1 - Tr\left(\hat{\rho}'^2\right)\right]}$$

where $|\tilde{\psi}\rangle$ represents the spin flip operator and is obtained by $|\tilde{\psi}\rangle = \sigma_y \otimes \sigma_y|\psi^*\rangle$ and $|\psi^*\rangle$ is the complex conjugate of ψ and where $\hat{\rho}'^2 = Tr_2\hat{\rho}$ is reduced density matrix. For mixed state

$$C(\hat{\rho}) = max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\}$$

where $\mu_i's$ are the square roots of the eigenvalues of $\hat{\rho}\tilde{\rho}$ in descending order when it has no more than two non zero eigen values of operator $\hat{\rho}\tilde{\rho}$. Here

$$\tilde{\rho} = \left(\sigma_y \otimes \sigma_y\right)\hat{\rho}^*\left(\sigma_y \otimes \sigma_y\right).$$

Next section deals with the new proposal of entanglement measure. It is also known as computational Cross norm Criterion (CCN) and in this paper, an attempt is made to show that it may be used to quantify entanglement.

III. METHODOLOGY

The inductive reasoning methodology has been adopted for the research. It aims to engender meanings from the prepared set in order to identify patterns and connectivity to build a theory patterns, resemblance and regularities have been observed in order to reach conclusion. MATLAB programming is used to calculate values of the two measures.

IV. FINDINGS AND SUGGESTION

Here we suggest similar to negativity, the following as entanglement measure

$$N_R(\hat{\rho}) = \|\hat{\rho}^R\|_{tr} - 1,$$

as entanglement measure. This measure is based upon realignment of a density matrix. For separable states, $\|\hat{\rho}^R\|_{tr} = 1, N_R(\hat{\rho}) = 0$ and for Bell state $\|\hat{\rho}^R\|_{tr} = 2, N_R(\hat{\rho}) = 1$. It is observed that, this measure and concurrence both are equivalent for certain states. Evaluation of $N_R(\hat{\rho})$ is much simpler than the concurrence as well as it detects entanglement simultaneously.
A comparative study of these two criteria is given as under:

A. *For Werner States:* The density matrix for Werner state is given by

$$\hat{\rho} = \frac{p}{2}(|00\rangle\langle 00| + |11\rangle\langle 11| \pm|00\rangle\langle 11| \pm |11\rangle\langle 00|) + \frac{(1-p)}{4}(|00\rangle\langle 00| + |11\rangle\langle 11| + |01\rangle\langle 01| + |10\rangle\langle 10|)$$

For different values of $p > \frac{1}{3}$, corresponding $N_R(\hat{\rho})$ is tabulated with concurrence and NR=$N_R(\hat{\rho})$ see table 1.

B. For mixture of Bell States: The density matrix will be

$$\hat{\rho} = p|\phi^{\pm}\rangle\langle\phi^{\pm}| + (1-p)|\psi^{\pm}\rangle\langle\psi^{\pm}|$$

TABLE I FOR WERNER STATES

S. No.	P	NR	Concurrence
1	0.333333	0	0
2	0.4	0.1	0.1
3	0.5	0.25	0.25
4	0.6	0.4	0.4
5	0.7	0.55	0.55
6	0.8	0.7	0.7
7	0.9	0.85	0.85
8	1	1	1

TABLE II FOR MIXTURE OF BELL STATES

S. No.	P	NR	Concurrence
1	0.5	0	0
2	0.6, 0.4	0.2	0.2
3	0.7, 0.3	0.4	0.4
4	0.8, 0.2	0.6	0.6
5	0.9,0 .1	0.8	0.8

C. For Mixture of Bell States and Separable States: Its density matrix will be

$$\hat{\rho} = p|\phi^{+}\rangle\langle\phi^{+}| + (1-p)|\chi\rangle\langle\chi|,$$

Where$|\chi\rangle = \frac{1}{4}(|00\rangle + |11\rangle + |01\rangle + |10\rangle)$.

The concurrence and $N_R(\hat{\rho})$of state under consideration has been computed and are tabulated below.

V. ANALYSIS

Since negativity is based on partial transposition of density matrix it triggers to think whether the realignment criterion may be used to measure entanglement. It is compared with the well-established entanglement measure concurrence in which density matrix is manipulated under the multiple operation of σ_y and its cross product in a systematic manner. The results found are in good agreement.

TABLE III FOR MIXTURE OF BELL STATES AND SEPARABLE STATES

S. No.	P	NR	Concurrence
1	0.1	0.1	0.1
2	0.2	0.2	0.2
3	0.3	0.3	0.2999
4	0.4	0.4	0.4
5	0.5	0.5	0.4999
6	0.6	0.6	0.6
7	0.7	0.7	0.7
8	0.8	0.8	0.7999
9	0.9	0.9	0.8996

VI. CONCLUSION

It is evident that the proposed entanglement measure and the established one i.e. concurrence are equivalent for states under consideration. It may be conjectured that this may be used to quantify entanglement in other states too. Computation of concurrence for pure state is easy but difficult for mixed state and therefore it may find broad spectrum of applicability.

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