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Edge Waves in an Initially Stressed Visco-Poroelastic Plate under Plane Stress Condition

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***Abstract* - The purpose of this paper is to investigate the propagation of edge waves in a homogeneous visco-poroelastic plate which is initially stressed in horizontal direction. The pertinent governing equations are derived and the frequency equation is obtained in the framework of Biot's theory. Frequency and attenuation are computed as a function of wavenumber. For the numerical process, solids namely, sandstone saturated with kerosene, sandstone saturated with water is considered and the results are presented graphically.**
***Keywords:* Visco-Poroelastic Plate, Initial Stress, Frequency, Attenuation.**

I. INTRODUCTION

Wave propagation problems in the visco-poroelastic solids have many applications in Engineering, Biomechanics, and Geophysics. Mindlin (1986) studied flexural vibrations of rectangular plates with free edges. Local and global damped vibrations of plates with a viscoelastic soft flexible core are investigated by Malekzadeh *et al.*,(2005).The plane wave at the edge of a uniformly pre-stressed fiber-reinforced plate is discussed by Abhishek *et al.*,(2014). In this paper, it is found that the edge wave does not propagate in a very thick/thin isotropic plate under no initial stress. Edge waves in an initially stressed visco-elastic plate are investigated by Nirmala *et al*., (2015). Nonlinear vibrations of viscoelastic rectangular plates are studied by Amabili (2016). Santimoy and Manisha (2017) studied edge wave propagation in an initially stressed dry sandy plate. In the paper, the dispersion relation is found to be effected by the initial stress and sandiness parameters present in the plate.

Different approaches have been used to model poroelastic solids in the framework of Biot's theory (1956). Rayleigh and Stonely waves in poroelastic half-space and a porous solid lying over elastic solid are examined by Tajuddin *et al.*,(1984,1990). Dynamic interaction of a poroelastic layer and half-space is discussed by Tajuddin and Ahmed (1991). Edge waves in poroelastic plates under plane stress conditions are investigated by Malla Reddy and Tajuddin (2003). They derived constitutive relations and governing equations of motion under plane stress conditions. Three dimensional vibration analysis of an infinite poroelastic plate immersed in an inviscid elastic plate is studied by Shah and Tajuddin (2011).

Neelam and Miglani (2013) analyzed plane strain deformation of a poroelastic half space in welded contact with an isotropic elastic half space. Dispersion study of plane strain vibrations in poroelastic solid bars with polygonal cross section is studied by Sandhya *et al.*, (2015). In the said paper, Fourier collocation method is used to solve the plane strain problem and the frequency equations in the case of symmetric and anti-symmetric modes are discussed. However, to the best of author's knowledge, plane strain vibrations of visco-poroelastic plates are not yet investigated. Therefore, in this paper, the same is investigated in the framework of Biot's theory. The pertinent governing equations are derived in presence of initial stress. Frequency and attenuation are computed as a function of wavenumber. For numerical process, two types of materials are considered and then discussed.

The rest of the paper is organized as follows. In section 2, formulation and solution of the problem is given. Numerical results are presented graphically in section 3. Finally, the conclusion is given in section 4.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Consider a visco-poroelastic plate of thickness h and plate of semi-infinite length in rectangular coordinate system. The plate is horizontally initially stressed $(p=-\sigma_{xx})$ along the x - direction as shown in the fig. 1. Further assume that plate is under plane stress condition.

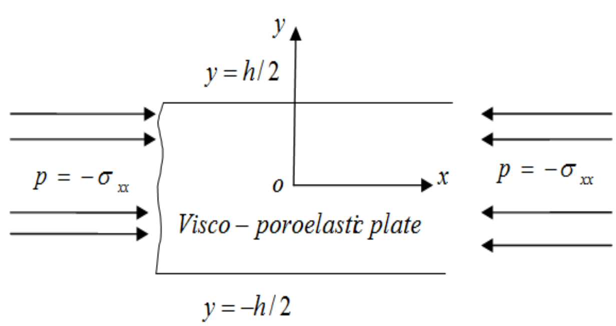

Fig. 1 Geometry of the problem

Let $\vec{u}(u,v,0)$ and $\vec{U}(U,V,0)$ be the solid and fluid displacements. The equations of motion for the propagation of waves are given under

$$P^*\frac{\partial^2 u}{\partial x^2}+(\overline{N}+\frac{p}{2})\frac{\partial^2 u}{\partial y^2}+(A^*+\overline{N}-\frac{p}{2})\frac{\partial^2 v}{\partial x\partial y}+Q^*(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U)+b\frac{\partial}{\partial t}(u-U),$$
$$(P^*-\frac{p}{2})\frac{\partial^2 v}{\partial y^2}+\overline{N}\frac{\partial^2 v}{\partial x^2}+(A^*+\overline{N}+\frac{p}{2})\frac{\partial^2 u}{\partial x\partial y}+Q^*(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 U}{\partial x\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V)+b\frac{\partial}{\partial t}(v-V),$$
$$Q^*(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y})+R^*(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 V}{\partial x\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U)-b\frac{\partial}{\partial t}(u-U),$$
$$Q^*(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2})+R^*(\frac{\partial^2 U}{\partial x\partial y}+\frac{\partial^2 V}{\partial y^2})=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V)-b\frac{\partial}{\partial t}(v-V),\tag{1}$$

In eq. (1), $p=-\sigma_{xx}$ is the horizontal initial stress along x-direction. b is the dissipative coefficient, and $\rho_{11},\rho_{12},\rho_{22}$ are mass coefficients. The notations e and ε are the dilatations of solid and fluid respectively. A^*,Q^*,R^* are modified poroelastic constants, and are given by (Malla Reddy and Tajuddin, 2003)

$$A^*=\frac{A(R+2N-Q^2}{A+2Q+R+2N},\ Q^*=\frac{Q(Q+2N)-AR}{A+2Q+R+2N},\ R^*=\frac{R(A+2N)-Q^2}{A+2Q+R+2N},$$

$\overline{N}=N+N'\frac{\partial}{\partial t}$, N is the shear modulus and N' is the viscosity. Assume that the solution to the eq. (1) in the frequency domain as follows:

$$u(x,y)=C_1e^{j\omega t-j(k_1x+k_2y)},$$
$$w(x,y)=C_2e^{j\omega t-j(k_1x+k_2y)},$$
$$U(x,y)=C_3e^{j\omega t-j(k_1x+k_2y)},$$
$$W(x,y)=C_4e^{j\omega t-j(k_1x+k_2y)}.\tag{2}$$

In the above, C_1,C_2,C_3,C_4 are arbitrary constants, j is the complex unity, k_i $(i=1,2)$ is the wavenumber in the i^{th} direction such that the wavenumber $k=\sqrt{k_1^2+k_2^2}$. Substituting the eq. (2) in the eq. (1), the equations of motion in terms of displacements are obtained as follows:

$$C_1(P^*k_1^2+(N+N'j\omega+\frac{p}{2})k_2^2+\omega^2(\rho_{11}-\frac{jb}{\omega}))-C_2(A^*+(N+N'j\omega-\frac{p}{2})k_1k_2)$$
$$-C_3(Q^*k_1^2-\omega^2(\rho_{12}+\frac{jb}{\omega}))-C_4(Q^*k_1k_2)=0,$$
$$C_1((-(N+N'j\omega)k_2^2)-(A^*+(N+N'j\omega+\frac{p}{2})k_1k_2))+C_2((-P^*+\frac{p}{2})k_2^2+\omega^2(\rho_{11}-\frac{jb}{\omega}))$$
$$-C_3(Q^*k_1k_2)-C_4((Q^*k_2^2)-\omega^2(\rho_{12}+\frac{jb}{\omega}))=0,$$
$$C_1(-Q^*k_1^2+\omega^2(\rho_{12}-\frac{jb}{\omega}))+C_2(Q^*k_1k_2)+C_3(-R^*k_1^2+\omega^2(\rho_{22}+\frac{jb}{\omega}))-C_4(R^*k_1k_2)=0,$$
$$C_1(-Q^*k_1k_2)+C_2(-Q^*k_2^2+\omega^2(\rho_{12}-\frac{jb}{\omega}))+C_3(-R^*k_1k_2)+C_4(-R^*k_2^2+\omega^2(\rho_{22}+\frac{jb}{\omega}))=0.\tag{3}$$

III. NUMERICAL RESULTS

For the numerical work, the wave propagation is considered along $y-$direction. In this case $k_1=0$. Then, the equation of motion reduces to the following form.

$$[A_{lm}][C_l]=0,\ l=m=1,2,3,4,\tag{4}$$

where $A_{11}=(N+N'j\omega+\frac{p}{2})k_2^2+\omega^2(\rho_{11}-\frac{jb}{\omega}),$

$$A_{13}=\omega^2(\rho_{12}+\frac{jb}{\omega}),$$
$$A_{22}=(-P^*+\frac{p}{2})k_2^2+\omega^2(\rho_{11}-\frac{jb}{\omega}),$$
$$A_{24}=(-Q^*k_2^2+\omega^2(\rho_{12}+\frac{jb}{\omega})),$$
$$A_{31}=\omega^2(\rho_{12}-\frac{jb}{\omega}),\ A_{33}=\omega^2(\rho_{22}+\frac{jb}{\omega}),$$
$$A_{42}=(-Q^*k_2^2+\omega^2(\rho_{12}-\frac{jb}{\omega})),$$
$$A_{44}=(-R^*k_2^2+\omega^2(\rho_{22}+\frac{jb}{\omega})),$$
$$A_{12}=A_{14}=A_{21}=A_{23}=A_{32}=A_{34}=A_{41}=A_{43}=0.\tag{5}$$

Eq. (4) results in a system of four homogeneous equations in four arbitrary constants C_1,C_2,C_3,C_4. For a non trivial solution, determinant of coefficients is zero. Accordingly, the following frequency equation is obtained.

$$\left|c_{lm}\right| + i\left|d_{lm}\right| = 0 \quad (l, m = 1, 2, 3, 4). \tag{6}$$

The expressions for c_{lm} and d_{lm} in eq. (6) are similar as that of eq. (5), after separating the real and imaginary parts.The complex valued frequency eq. (6) gives implicit relation between the frequency, attenuation, and wavenumber. Attenuation coefficient (Q^{-1}) is computed by using $Q^{-1} = \frac{2\,\mathrm{Im}\,(\omega)}{\mathrm{Re}\,(\omega)}$. For the illustration purpose, two types of poroelastic solids are used, namely, sandstone saturated with kerosene (Material I, Say) (Fatt, 1957), sandstone saturated with water (Material-II, Say) (Yew and Jogi, 1976). The physical parameter values of these materials values are given in the Table I.

TABLE I MATERIAL PARAMETERS

Material Parameters	A (N/m^2)	N (N/m^2)	Q (N/m^2)	R (N/m^2)	ρ_{11} (kg/m^3)	ρ_{12} (kg/m^3)	ρ_{22} (kg/m^3)	N' (kg/m^3)
Material-I	0.443×10^{10}	0.276×10^{10}	0.076×10^{10}	0.032×10^{10}	1.926×10^{3}	-0.002×10^{3}	0.215×10^{3}	1.64×10^{-3}
Material-II	0.306×10^{10}	0.922×10^{10}	0.013×10^{10}	0.063×10^{10}	1.903×10^{3}	0	0.226×10^{3}	1.00×10^{-3}

The dissipative coefficient is taken to be 0.17×10^3 N/m^2 as in the paper (Malla Reddy and Sandhya Rani, 2016). Initial stress value is taken to be '2' arbitrarily. Employing these values in eq. (6), the frequency and attenuation coefficient are computed as a function of the wavenumber. The values are computed using bisection method implemented in MATLAB, and the results are depicted in fig. 2 and 3. From the fig. 2, it is seen that as wavenumber increases, frequency, in general, increases in the case of Material-I and Material-II.

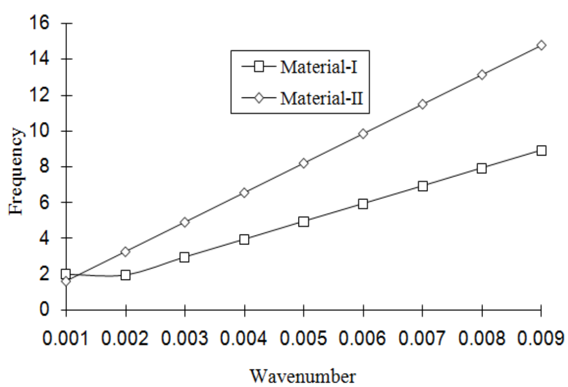

Fig. 2 Variation of frequency with wavenumber

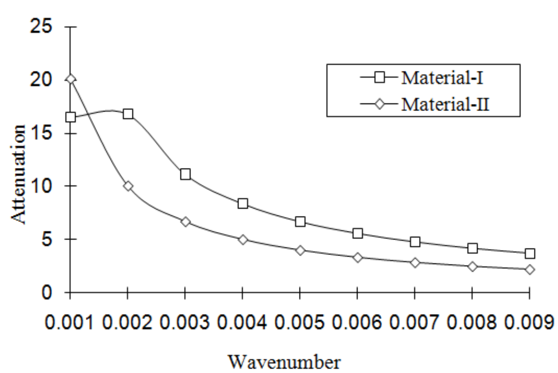

Fig. 3 Variation of attenuation with wavenumber

Also it is clear that frequency of Material-II values are higher than that of Material-I. From the fig. 3, it is clear that as wavenumber increases, in general, attenuation decreases in the case of Material-I and Material-II. Attenuation values of Material-I are higher than that of Material-II. Moreover, for fixed wavenumber, frequency and attenuation are computed against initial stress, and the values obtained are the same for all values of initial stresses. Therefore, in this case, it is concluded that frequency and attenuation are independent of initial stress.

IV. CONCLUSION

Employing Biot's theory, propagation of edge waves in a pre-stressed homogeneous visco-poroelastic plate is investigated. Frequency and attenuation are computed for two types of materials. For both the materials, as wavenumber increases, frequency, in general increases and attenuation, in general decreases. Similar analysis can be made for different poroelastic solids if pertinent values are available.

V. ACKNOWLEDGEMENT

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